Mathematica Pannonica
26/2 (2017–2018), 67–78

Yamabe solitons on generalized (k,μ)-space-forms

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Received: 2018. January 30

MSC 2010: 53 C 15, 53 C 25, 53 C 40.

Keywords: Yamabe soliton, generalized (k,μ)-space-form, semisymmetric metric connection.

Abstract: The object of the present paper is to study of Yamabe solitons on generalized (k,μ)-space-forms with respect to semisymmetric metric connection and obtained sufficient conditions for which such Yamabe soliton turns out to be a Yamabe soliton with respect to Levi-Civita connection.

1. Introduction

The notion of Yamabe flow was introduced by Hamilton ([9], [10]) as a tool for constructing metrics of constant scalar curvature in a given conformal class of Riemannian metrics on a Riemannian manifold (M^n, g), $n \geq 3$. The Yamabe flow is an evolution equation for metrics on a Riemannian manifold as follows:

$$\frac{\partial}{\partial t} g = -rg,$$

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where r is the scalar curvature corresponds to g. In dimension $n = 2$, the Yamabe flow is equivalent to the Ricci flow. However, in dimension $n > 2$, the Yamabe and Ricci flows do not agree as the first one preserves the conformal class of the metric but the Ricci flow does not in general.

A Yamabe soliton is a special solution of the Yamabe flow that moves by one parameter family of diffeomorphisms generated by a fixed (time-independent) vector field V on M and homothetic.

A Yamabe soliton on a Riemannian manifold (M, g) is a triplet (g, V, σ) such that

$$\frac{1}{2}\pounds_V g = (r - \sigma)g, \tag{1.1}$$

where $\pounds_V$ denotes the Lie derivative in the direction of the vector field V and σ is a constant. The Yamabe soliton is said to be shrinking, steady and expanding according as $\sigma < 0, = 0$ and > 0 respectively. If σ is a smooth function on M then the metric satisfying (1.1) is called almost Yamabe soliton [2]. It may be noted that Yamabe solitons coincide with the Ricci solitons in dimension $n = 2$ and for $n > 2$, the Ricci solitons and Yamabe solitons have different behaviours.

On the analogy of (k, μ)-contact metric manifold [4], a contact metric manifold M is said to be a generalized (k, μ)-space [5] if its curvature tensor R satisfies the condition

$$R(X, Y)\xi = k\{\eta(Y)X - \eta(X)Y\} + \mu\{\eta(Y)hX - \eta(X)hY\} \tag{1.2}$$

for some smooth functions k and μ on M which are independent of the choice of vector fields X and Y. If k and μ are constants then the manifold M is called a (k, μ)-space.

A (k, μ)-space M of dimension greater than 3 with constant φ-sectional curvature c is called (k, μ)-space-form [12] and its curvature tensor R is given by [12]

$$\begin{aligned} R(X,Y)Z &= \frac{c+3}{4}R_1(X,Y)Z + \frac{c-1}{4}R_2(X,Y)Z \\ &+ \left(\frac{c+3}{4} - k\right)R_3(X,Y)Z + R_4(X,Y)Z + \frac{1}{2}R_5(X,Y)Z \\ &+ (1-\mu)R_6(X,Y)Z, \end{aligned} \tag{1.3}$$

where $R_1, R_2, R_3, R_4, R_5, R_6$ are defined as

$$R_1(X, Y)Z = g(Y, Z)X - g(X, Z)Y,$$

$$R_2(X,Y)Z = g(X,\varphi Z)\varphi Y - g(Y,\varphi Z)\varphi X + 2g(X,\varphi Y)\varphi Z,$$

$$R_3(X,Y)Z = \eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi,$$

$$R_4(X,Y)Z = g(Y,Z)hX - g(X,Z)hY + g(hY,Z)X - g(hX,Z)Y,$$

$$R_5(X,Y)Z = g(hY,Z)hX - g(hX,Z)hY + g(\varphi hX,Z)\varphi hY - g(\varphi hY,Z)\varphi hX,$$

$$R_6(X,Y)Z = \eta(X)\eta(Z)hY - \eta(Y)\eta(Z)hX + g(hX,Z)\eta(Y)\xi - g(hY,Z)\eta(X)\xi$$

for all vector fields X, Y, Z on M, where $h = \frac{1}{2}\pounds_\xi\varphi$. As a generalization of (k,μ)-space-form, in [6] Carriazo et al. introduced and studied the notion of generalized (k,μ)-space-form with the existence of such notion by several interesting examples. An almost contact metric manifold $M(\varphi,\xi,\eta,g)$ is called generalized (k,μ)-space-form [6] if there exist $f_1, f_2, f_3, f_4, f_5, f_6 \in C^\infty(M)$, the ring of smooth functions on M, such that

$$R(X,Y)Z = (f_1R_1 + f_2R_2 + f_3R_3 + f_4R_4 + f_5R_5 + f_6R_6)(X,Y)Z, \tag{1.4}$$

where R_1, R_2, R_3, R_4, R_5 and R_6 are defined as in (1.3) and such a manifold of dimension $(2n+1)$, $n > 1$ (the condition $n > 1$ is assumed throughout the paper), is denoted by $M(f_1, f_2, \cdots, f_6)$.

If, in particular, $f_1 = \frac{c+3}{4}, f_2 = \frac{c-1}{4}, f_3 = \frac{c+3}{4} - k, f_4 = 1, f_5 = \frac{1}{2}$ and $f_6 = 1 - \mu$ then the generalized (k,μ)-space-forms turns into the notion of (k,μ)-space-forms. In this connection it may be noted that the generalized (k,μ)-space-form is the generalization of the generalized Sasakian-space-forms introduced by Alegre et al. [1]. The generalized (k,μ)-space-forms have been also studied by Hui et al. ([11], [13]).

In [7] Friedmann and Schouten introduced the notion of semisymmetric linear connection on a differentiable manifold. Then in 1932 Hayden [8] introduced the idea of metric connection with torsion on a Riemannian manifold. A systematic study of the semisymmetric metric connection on a Riemannian manifold has been given by Yano in 1970 [15].

A linear connection $\bar{\nabla}$ in an n-dimensional differentiable manifold M is said to be a semisymmetric connection [15] if its torsion tensor τ of the connection $\bar{\nabla}$ is of the form

$$\tau(X,Y) = \bar{\nabla}_X Y - \bar{\nabla}_Y X - [X,Y] \tag{1.5}$$

satisfies

$$\tau(X,Y) = \eta(Y)X - \eta(X)Y,$$

where η is a 1-form. Again, if the semisymmetric connection $\bar{\nabla}$ satisfies the condition

$$(\bar{\nabla}_X g)(Y,Z) = 0 \tag{1.6}$$

for all X, Y, $Z \in \chi(M)$, where $\chi(M)$ is the Lie algebra of vector fields on the manifold M, then $\bar{\nabla}$ is said to be a semisymmetric metric connection. Semisymmetric metric connection have been studied by many authors in several ways to a different extent. The semisymmetric connection $\bar{\nabla}$ in a generalized (k,μ)-space-form $M(f_1, f_2, \cdots, f_6)$ is defined by [14]

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi, \tag{1.7}$$

where ∇ is the Levi-Civita connection on M.

The object of the present paper is to study Yamabe solitons on generalized (k,μ)-space-forms with respect to semisymmetric metric connection. The paper is structured as follows. Section 2 is concerned with preliminaries. Section 3 deals with the study of Yamabe solitons on generalized (k,μ)-space-forms with respect to Levi-Civita and semisymmetric metric connection. It is shown that if (g,ξ,σ) is a Yamabe soliton on a generalized (k,μ)-space-form then its scalar curvature is constant and this soliton is shrinking, steady and expanding depending upon the sign of the scalar curvature. The Yamabe soliton (g,ξ,σ) with potential vector field ξ as torse forming on generalized (k,μ)-space-form is also studied. Also we found the sufficient condition of a Yamabe soliton on a generalized (k,μ)-space-form with respect to semisymmetric metric connection to be a Yamabe soliton on a generalized (k,μ)-space-form with respect to Levi-Civita connection. The Yamabe soliton on generalized (k,μ)-space-form whose potential vector field is pairwise collinear with Reeb vector field is also studied.

2. Preliminaries

An odd dimensional smooth manifold M is said to be an almost contact metric manifold [3] if there exist a (1,1) tensor field φ, a vector field ξ, a 1-form η and a Riemannian metric g on M such that

$$\varphi^2(X) = -X + \eta(X)\xi, \quad \varphi\xi = 0, \tag{2.1}$$

$$\eta(\xi) = 1, \quad g(X,\xi) = \eta(X), \quad \eta(\varphi X) = 0, \tag{2.2}$$

(2.3) $\quad g(\varphi X, \varphi Y) = g(X, Y) - \eta(X)\eta(Y), \quad g(\varphi X, Y) = -g(X, \varphi Y)$

for any vector fields X and Y on M. Such a manifold is said to be a *contact metric manifold* [3] if $d\eta(X, Y) = g(X, \varphi Y)$ for all $X, Y \in \chi(M)$.

Given a contact metric manifold $M^{2n+1}(\varphi, \xi, \eta, g)$, we define a (1,1) tensor field h by $2h = \pounds_\xi \varphi$. Then h is symmetric and satisfies the following relations

(2.4) $\quad h\xi = 0, \quad h\varphi = -\varphi h, \quad tr(h) = tr(\varphi h) = 0, \quad \eta(hX) = 0$

for all $X \in \chi(M)$.

Moreover, if ∇ denotes the Riemannian connection of g, then the following relation holds:

(2.5) $\quad \nabla_X \xi = -\varphi X - \varphi hX, \ (\nabla_X \eta)(Y) = g(X + hX, \varphi Y).$

In a $(2n + 1)$-dimensional (k, μ)-contact metric manifold, we have [4]

(2.6) $$h^2 = (k - 1)\varphi^2, \quad k \leq 1,$$

(2.7) $$(\nabla_X \varphi)(Y) = g(X + hX, Y)\xi - \eta(Y)(X + hX),$$

(2.8)
$$(\nabla_X h)(Y) = [(1-k)g(X, \varphi Y)+g(X, h\varphi Y)]\xi+\eta(Y)h(\varphi X+\varphi hX)-\mu\eta(X)\varphi hY.$$

For an almost contact metric manifold, a φ-section of M at $p \in M$ is a section $\pi \subseteq T_pM$ spanned by a unit vector X_p orthogonal to ξ_p and φX_p. The φ-sectional curvature of π is defined by $K(X \wedge \varphi X) = g(R(X, \varphi X)\varphi X, X)$. A (k, μ)-space of dimension greater than 3 with constant φ-sectional curvature c is called a *(k, μ)-space-form.*

In a generalized (k, μ)-space-form, we have ([1],[6], [14])

(2.9)
$$R(X, Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\},$$

(2.10)
$$R(\xi, Y)Z = (f_1 - f_3)\big[g(Y, Z)\xi - \eta(Z)Y\big] + (f_4 - f_6)\big[g(hY, Z)\xi - \eta(Z)hY\big].$$

(2.11)
$$QX = (2nf_1 + f_2 - f_3)X + [(2n - 1)f_4 - f_6)]hX - [3f_2 + (2n - 1)f_3)]\eta(X)\xi,$$

$$S(X,Y) = (2nf_1 + f_2 - f_3)g(X,Y) + [(2n-1)f_4 - f_6]g(hX,Y) - [3f_2 + (2n-1)f_3)]\eta(X)\eta(Y), \quad (2.12)$$

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X), \quad (2.13)$$

$$r = 2n[(2n+1)f_1 + 3f_2 - f_3] \quad (2.14)$$

for any X, Y, Z, $\in \chi(M)$ where Q is the Ricci operator and S is the Ricci tensor of $M(f_1, f_2, \cdots, f_6)$.

A vector field ξ is called a torse-forming vector field [16] on a generalized (k,μ)-space-form if $\nabla_X \xi = \rho X + \gamma(X)\xi$, where ρ is a smooth function and γ is a nowhere vanishing 1-form.
Further, if $\bar{R}$ is the curvature tensor, $\bar{S}$ is the Ricci tensor and $\bar{r}$ is the scalar curvature of $M(f_1, f_2, \cdots, f_6)$ with respect to semisymmetric metric connection then we have [14]

$$\bar{R}(X,Y)\xi = \left(f_1 - f_3 - \frac{1}{2}\right)\{\eta(Y)X - \eta(X)Y\} + (f_4 - f_6)\{\eta(Y)hX - \eta(X)hY\} - \eta(Y)\beta(X) + \eta(X)\beta(Y), \quad (2.15)$$

$$\begin{aligned} \bar{R}(\xi,Y)Z &= (f_1 - f_3 - 2)\{g(Y,Z)\xi - \eta(Z)Y\} \\ &+ (f_4 - f_6)\{g(hY,Z)\xi - \eta(Z)hY\} \\ &+ g(\varphi Y, Z)\xi + g(\varphi hY, Z)\xi - \eta(Z)[\varphi Y + \varphi hY], \end{aligned} \quad (2.16)$$

$$\begin{aligned} \eta(\bar{R}(X,Y)Z) &= \left(f_1 - f_3 - \frac{1}{2}\right)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} \\ &+ (f_4 - f_6)\{g(hY,Z)\eta(X) - g(hX,Z)\eta(Y)\} \\ &- \alpha(Y,Z)\eta(X) + \alpha(X,Z)\eta(Y), \end{aligned} \quad (2.17)$$

$$\bar{S}(X,\xi) = \left[2n(f_1 - f_3) - \frac{2n-1}{2} - trace(\alpha)\right]\eta(X), \quad (2.18)$$

$$\bar{S}(X,Y) = S(X,Y) - trace(\alpha)g(X,Y) - (2n-1)\alpha(X,Y) \quad (2.19)$$

and

$$\bar{r} = r - 4n\ trace(\alpha), \quad (2.20)$$

where $\alpha(X,Y) = g(\beta(X),Y)$ and $\beta(X) = \bar{\nabla}_X \xi + \frac{1}{2}X$ for all X, Y on $M(f_1, f_2, \cdots, f_6)$.

3. Yamabe solitons on generalized (k,μ)-space-form

Let (g,ξ,σ) be a Yamabe soliton on a generalized (k,μ)-space-form. Then we have from (1.1) that

$$\frac{1}{2}(\pounds_\xi g)(Y,Z) = (r-\sigma)g(Y,Z). \tag{3.1}$$

From (2.1)-(2.3) and (2.5) we have

$$(\pounds_\xi g)(Y,Z) = g(\nabla_Y \xi, Z) + g(Y, \nabla_Z \xi) = 0. \tag{3.2}$$

Using (3.2) in (3.1) we get $r = \sigma =$ constant and hence we can state the following:

Theorem 3.1. If (g,ξ,σ) is a Yamabe soliton on a generalized (k,μ)-space-form M then its scalar curvature is constant and the soliton is shrinking, steady and expanding according as $(2n+1)f_1+3f_2-f_3 < 0, = 0$ and > 0 respectively.

Corollary 3.2. If (g,ξ,σ) is a Yamabe soliton on a (k,μ)-space-form M then its scalar curvature is constant and the soliton is shrinking, steady and expanding according as $k + \frac{1}{4}[(2n+3)c + 6n - 3] < 0, = 0$ and > 0 respectively.

Remark 1: The Theorem 3.1 is also same for generalized Sasakian-space-form instead of generalized (k,μ)-space-form.

Corollary 3.3. If (g,ξ,σ) is a Yamabe soliton on a Sasakian-space-form then its scalar curvature is constant and the soliton is shrinking, steady and expanding according as $(2n+3)c + 6n + 1 < 0, = 0$ and > 0 respectively.

If ξ is torse-forming vector field on a generalized (k,μ)-space-form then by definition, we have

$$g(\nabla_X \xi, \xi) = (\rho\eta \ + \gamma)X$$

and hence by virtue of (2.5) it follows that $\gamma = -\rho n$. Thus we obtain

$$\nabla_X \xi = \rho\{X - \eta(X)\xi\}. \tag{3.3}$$

Using (3.3), we can compute

$$(\pounds_\xi g)(Y,Z) = 2\rho[g(Y,Z) - \eta(Y)\eta(Z)]. \tag{3.4}$$

In view of (2.14) and (3.4) it follows from (3.1) that

$$\rho[g(Y,Z) - \eta(Y)\eta(Z)] = [2n\{(2n+1)f_1 + 3f_2 - f_3\} - \sigma]g(Y,Z) \tag{3.5}$$

from which we get

$$\rho = (1 + \frac{1}{2n})[2n\{(2n+1)f_1 + 3f_2 + f_3\} - \sigma] \tag{3.6}$$

This leads to the following:

Theorem 3.4. If (g, ξ, σ) is a Yamabe soliton on a generalized (k, μ)-space-form (respectively generalized Sasakian-space-form) with potential vector field ξ as torce-forming then the smooth function ρ is given in *(3.6)*.

Now, Let us take (g, ξ, σ) be a Yamabe soliton on a generalized (k, μ)-space-form with respect to semisymmetric metric connection. Then we have

$$\frac{1}{2}(\bar{\pounds}_\xi g)(Y,Z) = (\bar{r} - \sigma)g(Y,Z), \tag{3.7}$$

where $\bar{\pounds}_\xi$ is the Lie derivative along the vector field ξ on M with respect to semisymmetric metric connection.
Again form (1.7), (2.1) - (2.3) and (2.5), we compute

$$\begin{aligned}(\bar{\pounds}_\xi g)(Y,Z) &= g(\bar{\nabla}_Y \xi, Z) + g(Y, \bar{\nabla}_Z \xi) \\ &= 2[g(Y,Z) - \eta(Y)\eta(Z)].\end{aligned} \tag{3.8}$$

Using (2.20) and (3.8) in (3.7) we get

$$[r - 4n\ trace(\alpha) - \sigma - 1]g(Y,Z) + \eta(Y)\eta(Z) = 0.$$

Contracting the above relation over Y and Z, we get
$r = \sigma + \frac{1}{n}(6n^3 - 4n^2 + n - 1) =$ constant and hence we can state the following:

Theorem 3.5. If (g, ξ, σ) is a Yamabe soliton on a generalized (k, μ)-space-form with respect to semisymmetric metric connection then its scalar curvature is constant and the soliton is shrinking, steady and expanding according as

$$(2n+1)f_1 + 3f_2 - f_3 \lesseqgtr \frac{1}{n}(6n^3 - 4n^2 + n - 1)$$

respectively.

Corollary 3.6. If (g, ξ, σ) is a Yamabe soliton on a (k, μ)-space-form with respect to semisymmetric metric connection then its scalar curvature is constant and the Yamabe soliton is shrinking, steady and expanding according as $k + \frac{1}{4}[(2n+3)c + 6n - 3] \lesseqgtr \frac{1}{n}(6n^3 - 4n^2 + n - 1)$ respectively.

Corollary 3.7. If (g, ξ, σ) is a Yamabe soliton on a Sasakian-space-form with respect to semisymmetric metric connection then its scalar curvature is constant and the Yamabe soliton is shrinking, steady and expanding according as $n[(2n+3)c + 6n + 1] \lesseqgtr 4[6n^3 - 4n^2 + n - 1]$ respectively.

We now consider (g, V, σ) is a Yamabe soliton on a generalized (k, μ)-space-form $M(f_1, f_2, \cdots, f_6)$ with respect to semisymmetric metric connection. Then we have

$$\frac{1}{2}(\bar{\pounds}_V g)(Y, Z) = (\bar{r} - \sigma)g(Y, Z) \tag{3.9}$$

where $\bar{\pounds}_V$ is the Lie derivative along the vector field V on M with respect to semisymmetric metric connection. By virtue of (1.7), we have

$$\begin{aligned}(\bar{\pounds}_V g)(Y, Z) &= g(\bar{\nabla}_Y V, Z) + g(Y, \bar{\nabla}_Z V) \\ &= (\pounds_V g)(Y, Z) + 2\eta(V)g(Y, Z) \\ &- [\eta(Z)g(Y, V) + \eta(Y)g(Z, V)].\end{aligned} \tag{3.10}$$

Using (2.20) and (3.10) in (3.9), we get

$$\begin{aligned}\frac{1}{2}(\pounds_V g)(Y, Z) &= (r - \sigma)g(Y, Z) - [2n(3n-2) + \eta(V)]g(Y, Z) \\ &+ \frac{1}{2}[\eta(Z)g(Y, V) + \eta(Y)g(Z, V)].\end{aligned} \tag{3.11}$$

Theorem 3.8. A Yamabe soliton (g, V, σ) be on a generalized (k, μ)-space-form $M(f_1, f_2, \cdots, f_6)$ is invariant under semisymmetric metric connection if and only if the relation

$$\{2n(3n-2) + \eta(V)\}g(Y, Z) = \frac{1}{2}[\eta(Z)g(Y, V) + \eta(Y)g(Z, V)]$$

holds for arbitrary vector fields Y and Z.

Let (g, V, σ) be a Yamabe soliton on a generalized (k, μ)-space-form $M(f_1, f_2, \cdots, f_6)$ with respect to semisymmetric metric connection such that V is pairwise collinear with ξ, i.e, $V = b\xi$, where b is a function. Then (3.9) holds, which implies by virtue of (2.20) and (3.8) that

$$\begin{aligned} b[g(Y, Z) - \eta(Y)\eta(Z)] &+ \frac{1}{2}(Yb)\eta(Z) + \frac{1}{2}(Zb)\eta(Y) \\ &= [r - 2n(3n-2) - \sigma]g(Y, Z). \end{aligned} \tag{3.12}$$

Putting $Z = \xi$ in (3.12) and using (2.1)-(2.2) we get

$$\frac{1}{2}(Yb) + \frac{1}{2}(\xi b)\eta(Y) = [r - 2n(3n-2) - \sigma]\eta(Y). \tag{3.13}$$

Again setting $Y = \xi$ in (3.13) and using (2.2) we obtain

$$(\xi b) = r - 2n(3n-2) - \sigma. \tag{3.14}$$

In view of (3.14), it follows from (3.13) that

$$db = [r - 2(3n-2) - \sigma]\eta. \tag{3.15}$$

Applying d on (3.15) we get

$$[r - 2n(3n-2) - \sigma]d\eta = 0. \tag{3.16}$$

Since $d\eta \neq 0$ we have from (3.16) that $r - 2n(3n-2) - \sigma = 0$ and hence from (3.15) that $db = 0$, which implies that b is constant. This leads to the following:

Theorem 3.9. If (g, V, σ) is a Yamabe soliton on a generalized (k, μ)-space-form $M(f_1, f_2, \cdots, f_6)$ with respect to semisymmetric metric connection such that V is pointwise collinear with ξ then V is a constant multiple of ξ and the Yamabe soliton is shrinking, steady and expanding according as $(2n+1)f_1 + 3f_2 - f_3 \lesseqqgtr 2n(3n-2)$ respectively.

Corollary 3.10. If (g, V, σ) is a Yamabe soliton on a (k, μ)-space-form M with respect to semisymmetric metric connection such that V is pointwise collinear with ξ then V is a constant multiple of ξ and the Yamabe soliton is shrinking, steady and expanding according as $k+\frac{1}{4}[(2n+3)c+6n-3] \lesseqqgtr 2n(3n-2)$ respectively.

Corollary 3.11. If (g, V, σ) is a Yamabe soliton on a Sasakian-space-form M with respect to semisymmetric metric connection such that V is pointwise collinear with ξ then V is a constant multiple of ξ and the Yamabe soliton is shrinking, steady and expanding according as $(2n+3)c + 6n + 1 \lesseqqgtr 8n(3n-2)$ respectively.

Remark 2: If (g, V, σ) is a Yamabe soliton on a generalized (k, μ)-space-form $M(f_1, f_2, \cdots, f_6)$ with respect to Levi-Civita connection such that V is pointwise collinear with ξ then V is a constant multiple of ξ and the Yamabe soliton is shrinking, steady and expanding according as $(2n+1)f_1 + 3f_2 - f_3 \lesseqqgtr 0$ respectively.

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