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ON THE W_2-CURVATURE TENSOR OF GENERALIZED SASAKIAN-SPACE-FORMS

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Abstract: The object of the present paper is to study generalized Sasakian-space-forms admitting W_2-curvature tensor. We consider generalized Sasakian-space-forms satisfying some conditions such as $W_2 \cdot S = 0$, $R(\xi, U) \cdot W_2 = 0$ and $W_2 \cdot R = 0$. We also construct an example of generalized Sasakian-space-form which is W_2-flat.

1. Introduction

It is well known that in differential geometry the curvature of a Riemannian manifold plays a basic role and the sectional curvatures of a manifold determine the curvature tensor R completely. A Riemannian manifold with constant sectional curvature c is called a real-space-form

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and its curvature tensor R satisfies the condition

$$R(X,Y)Z = c\{g(Y,Z)X - g(X,Z)Y\}. \tag{1.1}$$

Models for these spaces are the Euclidean spaces ($c = 0$), the spheres ($c > 0$) and the hyperbolic spaces ($c < 0$).

In contact metric geometry, a Sasakian manifold with constant ϕ-sectional curvature is called Sasakian-space-form and the curvature tensor of such a manifold is given by

$$\begin{aligned} R(X,Y)Z = {} & \frac{c+3}{4}\big\{g(Y,Z)X - g(X,Z)Y\big\}+ \\ & + \frac{c-1}{4}\big\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big\}+ \\ & + \frac{c-1}{4}\big\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X+ \\ & + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\big\}. \end{aligned} \tag{1.2}$$

These spaces can also be modeled depending on $c > -3$, $c = -3$ or $c < -3$.

As a generalization of Sasakian-space-form, in [1] Alegre, Blair and Carriazo introduced and studied the notion of generalized Sasakian-space-form with the existence of such notions by several interesting examples. An almost contact metric manifold $M(\phi,\xi,\eta,g)$ is called generalized Sasakian-space-form if there exist three functions f_1, f_2, f_3 on M such that [1]

$$\begin{aligned} R(X,Y)Z = {} & f_1\big\{g(Y,Z)X - g(X,Z)Y\big\}+ \\ & + f_2\big\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big\}+ \\ & + f_3\big\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X+ \\ & + g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\big\} \end{aligned} \tag{1.3}$$

for all vector fields X, Y, Z on M, where R is the curvature tensor of M and such a manifold of dimension $(2n+1)$, $n > 1$ (the condition $n > 1$ is assumed throughout the paper), is denoted by $M^{2n+1}(f_1, f_2, f_3)$.

If, in particular, $f_1 = \frac{c+3}{4}$, $f_2 = f_3 = \frac{c-1}{4}$ then the generalized Sasakian-space-forms reduces to the notion of Sasakian-space-forms. But it is to be noted that generalized Sasakian-space-forms are not merely generalization of Sasakian-space-forms. It also contains a large class of almost contact manifolds. For example it is known that [3] any three dimensional (α, β)-trans Sasakian manifold with α, β depending on ξ is a generalized Sasakian-space-form. However, we can find generalized Sasakian-space-forms with non-constant functions and arbitrary dimensions.

The generalized Sasakian-space-forms have been studied by several authors such as Alegre and Carriazo ([2], [3], [4]), Belkhelfa, Deszcz and Verstraelen [5], Carriazo [7], Cîrnu [8], De and Sarkar ([10], [11]), Ghefari, Solamy and Shahid [12], Gherib, Gorine and Belkhelfa [13], Kim [14], Narain, Yadav and Dwivedi [16], Olteanu ([17], [18]), Shukla and Chaubey [27], Sular and Özgür [28], Yadav, Suthar and Srivastava [32] and many others. In [14] Kim studied conformally flat generalized Sasakian-space-forms. Also in [10] De and Sarkar studied the Weyl projective curvature tensor of generalized Sasakian-space-forms.

In 1970 Pokhariyal and Mishra [23] introduced new tensor fields, called W_2 and E tensor fields, in a Riemannian manifold and studied their properties. According to them a W_2-curvature tensor on a manifold $(M^{2n+1}, g), n > 1$, is defined by [23]

$$W_2(X,Y)Z = R(X,Y)Z + \frac{1}{2n}\big[g(X,Z)QY - g(Y,Z)QX\big], \tag{1.4}$$

where Q is the Ricci-operator, i.e., $g(QX,Y) = S(X,Y)$ for all X, Y. In this connection it may be mentioned that Pokhariyal and Mishra ([23], [24]) and Pokhariyal [19] introduced some new curvature tensors defined on the line of Weyl projective curvature tensor.

The W_2-curvature tensor was introduced on the line of Weyl projective curvature tensor and by breaking W_2 into skew-symmetric parts the tensor E has been defined. Rainich conditions for the existence of the non-null electrovariance can be obtained by W_2 and E, if we replace the matter tensor by the contracted part of these tensors. The tensor E enables to extend Pirani formulation of gravitational waves to Einstein space ([21], [22]). It is shown that [23] except the vanishing of complexion vector and property of being identical in two spaces which are in geodesic correspondence, the W_2-curvature tensor possesses the properties almost similar to the Weyl projective curvature tensor. Thus we can very well use W_2-curvature tensor in various physical and geometrical spheres in place of the Weyl projective curvature tensor.

The W_2-curvature tensor have also been studied by various authors in different structures such as De and Sarkar [9], Matsumoto, Ianus and

Mihai [15], Pokhariyal ([20], [21], [22]), Shaikh, Jana and Eyasmin [25], Shaikh, Matsuyama and Jana [26], Taleshian and Hosseinzadeh [29], Tripathi and Gupta [30], Venkatesha, Bagewadi and Kumar [31], Yildiz and De [33] and many others.

Motivated by the above studies, the object of the present paper is to study W_2-curvature tensor field in a generalized Sasakian-space-form. In this paper we have obtained many interesting results out of which some results are similar in case of the study of Weyl projective curvature tensor and some results are not. The paper is organized as follows. Sec. 2 is concerned with preliminaries. Sec. 3 is devoted to the study of W_2-flat generalized Sasakian-space-forms and obtain a necessary and sufficient condition for a generalized Sasakian-space-form to be W_2-flat. In Sec. 4, we study generalized Sasakian-space-form satisfying the condition $W_2 \cdot S = 0$. In Sec. 5, we study W_2-semisymmetric generalized Sasakian-space-forms and in this case, it is proved that either $f_1 = f_3$ or the curvature tensor R satisfies a definite condition. The last section deals with generalized Sasakian-space-form satisfying $W_2 \cdot R = 0$. It is shown that if a generalized Sasakian-space-form satisfies $W_2 \cdot R = 0$ then either the manifold is W_2-flat or the curvature tensor R of the manifold satisfies a definite condition.

2. Preliminaries

In an almost contact metric manifold, we have [6]

$$\phi^2(X) = -X + \eta(X)\xi, \phi\xi = 0, \tag{2.1}$$

$$\eta(\xi) = 1, g(X,\xi) = \eta(X), \eta(\phi X) = 0, \tag{2.2}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.3}$$

$$g(\phi X, Y) = -g(X, \phi Y), g(\phi X, X) = 0, \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(\nabla_X \xi, Y). \tag{2.5}$$

From (1.3) we have in a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$,

$$QX = (2nf_1 + 3f_2 - f_3)X - \{3f_2 + (2n-1)f_3\}\eta(X)\xi, \tag{2.6}$$

$$S(X,Y) = (2nf_1+3f_2-f_3)g(X,Y)-\{3f_2+(2n-1)f_3\}\eta(X)\eta(Y), \tag{2.7}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{2.8}$$

(2.9) $$R(X,Y)\xi = (f_1 - f_3)\{\eta(Y)X - \eta(X)Y\},$$

(2.10) $$R(\xi,X)Y = (f_1 - f_3)\{g(X,Y)\xi - \eta(Y)X\},$$

(2.11) $$\eta(R(X,Y)Z) = (f_1 - f_3)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\},$$

(2.12) $$S(X,\xi) = 2n(f_1 - f_3)\eta(X),$$

(2.13) $$S(\xi,\xi) = 2n(f_1 - f_3),$$

(2.14) $$W_2(X,Y)\xi = -\frac{1}{2n}\{3f_2 + (2n-1)f_3\}\{\eta(Y)X - \eta(X)Y\},$$

(2.15)
$$W_2(\xi,Y)Z = \frac{1}{2n}\{3f_2 + (2n-1)f_3\}\eta(Z)\{Y - \eta(Y)\xi\} = -W_2(Y,\xi)Z.$$

If an almost contact Riemannian manifold M satisfies the condition

(2.16) $$S = ag + b\eta \otimes \eta$$

for some functions a and b on M, then M is said to be an η-Einstein manifold.

If, in particular, $a = 0$ then this manifold will be called a special type of η-Einstein manifold.

3. W_2-flat generalized Sasakian-space-forms

Let us consider a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$, which is W_2-flat. Then from (1.4), we get

(3.1) $$R(X,Y)Z = \frac{1}{2n}\big[g(Y,Z)QX - g(X,Z)QY\big].$$

By virtue of (1.3) and (2.6), (3.1) yields

(3.2)
$$\begin{aligned}
&f_1\big\{g(Y,Z)X - g(X,Z)Y\big\}+\\
&+ f_2\big\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\big\}+\\
&+ f_3\big\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X+\\
&+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\big\} =\\
&= \frac{1}{2n}\big\{(2nf_1 + 3f_2 - f_3)\{g(Y,Z)X - g(X,Z)Y\}-\\
&- \{3f_2 + (2n-1)f_3\}\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\}\xi\big\}.
\end{aligned}$$

Replacing X by ϕX in (3.2), we get

$$\begin{aligned}(3.3)\qquad & f_1\big\{g(Y,Z)\phi X - g(\phi X,Z)Y\big\}+\\ &+ f_2\big\{g(\phi X,\phi Z)\phi Y - g(Y,\phi Z)\phi^2 X + 2g(\phi X,\phi Y)\phi Z\big\}+\\ &+ f_3\big\{g(\phi X,Z)\eta(Y)\xi - \eta(Y)\eta(Z)\phi X\big\} =\\ &= \frac{1}{2n}\big\{(2nf_1+3f_2-f_3)\{g(Y,Z)\phi X - g(\phi X,Z)Y\}+\\ &+ \{3f_2+(2n-1)f_3\}g(\phi X,Z)\eta(Y)\xi\big\}.\end{aligned}$$

Setting $Z=\xi$ in (3.3), we obtain

$$(3.4)\qquad \{3f_2+(2n-1)f_3\}\eta(Y)\phi X = 0.$$

Again putting $Y=\xi$ in (3.4), we get

$$\{3f_2+(2n-1)f_3\}\phi X = 0,$$

which implies that

$$(3.5)\qquad f_3 = \frac{3f_2}{1-2n}$$

as ϕX is not identically zero, in general.

Conversely, we now consider the relation (3.5) holds. Now in view of (1.3) and (2.7), we have from (1.4) that

$$\begin{aligned}(3.6)\qquad & \tilde{W}_2(X,Y,Z,U) =\\ &= f_2\big\{g(X,\phi Z)g(\phi Y,U) - g(Y,\phi Z)g(\phi X,U) + 2g(X,\phi Y)g(\phi Z,U)\big\}+\\ &+ f_3\big\{\eta(X)\eta(Z)g(Y,U) - \eta(Y)\eta(Z)g(X,U) + g(X,Z)\eta(Y)\eta(U)-\\ &- g(Y,Z)\eta(X)\eta(U) - g(X,Z)g(Y,U) + g(Y,Z)g(X,U)\big\},\end{aligned}$$

where $\tilde{W}_2(X,Y,Z,U) = g(W_2(X,Y)Z,U)$.

Substituting X by ϕX and Y by ϕY, we get from (3.6) that

$$\begin{aligned}(3.7)\qquad & \tilde{W}_2(\phi X,\phi Y,Z,U) =\\ &= f_2\big\{g(\phi X,\phi Z)g(\phi^2 Y,U) - g(\phi Y,\phi Z)g(\phi^2 X,U)+\\ &+ 2g(\phi X,\phi^2 Y)g(\phi Z,U)\big\}+\\ &+ f_3\big\{g(\phi Y,Z)g(\phi X,U) - g(\phi X,Z)g(\phi Y,U)\big\}.\end{aligned}$$

Let $\{e_i\}$ be an orthonormal basis of the tangent space at each point of the manifold. Then setting $Y = U = e_i$ in (3.7) and taking summation over i, $1 \leq i \leq 2n+1$, we obtain

$$\sum_{i=1}^{2n+1} \tilde{W}_2(\phi X, \phi e_i, Z, e_i) =$$
$$= f_2\Big\{\sum_{i=1}^{2n+1} g(\phi e_i, \phi e_i) g(\phi X, \phi Z) - g(\phi^2 Z, \phi^2 X)\Big\} +$$
$$+ f_3 g(\phi X, \phi Z).$$

Again contracting the last equation over X and Z and using (3.5), we get $f_2 = 0$ and hence $f_3 = 0$. Consequently we have from (1.3) that

$$R(X,Y)Z = f_1\{g(Y,Z)X - g(X,Z)Y\}. \tag{3.8}$$

Also from (3.8), we get

$$S(Y,Z) = 2nf_1 g(Y,Z), \tag{3.9}$$

i.e.,

$$QY = 2nf_1 Y. \tag{3.10}$$

By virtue of (3.8) and (3.10) we have from (1.4) that $W_2(X,Y)Z = 0$. Thus we can state the following:

Theorem 3.1. *Every generalized Sasakian-space-form* $M^{2n+1}(f_1, f_2, f_3)$ *is* W_2*-flat if and only if* $f_3 = \frac{3f_2}{1-2n}$.

This result is similar to the result as in case of projectively flat generalized Sasakian-space-form [10]. In [10] De and Sarkar proved that a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is projectively flat if and only if $f_3 = \frac{3f_2}{1-2n}$. Thus by virtue of above theorem we can state the following:

Theorem 3.2. *A generalized Sasakian-space-form* $M^{2n+1}(f_1, f_2, f_3)$ *is* W_2*-flat if and only if it is projectively flat.*

Again in [10] it is proved that a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ is projectively flat if and only if it is Ricci semisymmetric. By virtue of Th. 3.2, we can state the following:

Theorem 3.3. *A generalized Sasakian-space-form* $M^{2n+1}(f_1, f_2, f_3)$ *is* W_2*-flat if and only if it is Ricci semisymmetric.*

Corollary 3.1. *A Sasakian-space-form is W_2-flat if and only if $c = 1$.*

Every flat manifold is W_2-flat. However the converse is not true. If the manifold is W_2-flat then by virtue of (2.6) and (3.5) we have from (1.4) that

$$R(X,Y)Z = (f_1 - f_3)\{g(Y,Z)X - g(X,Z)Y\}.$$

Thus we can state the following:

Theorem 3.4. *Every flat generalized Sasakian-space-form is W_2-flat but the converse is true when $f_1 = f_3$.*

Example 3.1. Let $N(a,b)$ be a generalized complex space-form of dimension 4. Then $M = \mathbb{R} \times_f N$ endowed with the almost contact metric structure (ϕ, ξ, η, g_f) is a generalized Sasakian-space-form $M(f_1, f_2, f_3)$ of dimension 5 with

$$f_1 = \frac{a - f'^2}{f^2}, \quad f_2 = \frac{b}{f^2}, \quad f_3 = \frac{a - f'^2}{f^2} + \frac{f''}{f^2},$$

where f is a function of $t \in \mathbb{R}$ and f' denotes differentiation of f with respect to t [3]. We now choose f as a constant and $a = -b$. Then $f_3 = \frac{3f_2}{1-2.2}$ and $f_1 = f_3$. Consequently by virtue of Th. 3.1, we may conclude that the manifold M under consideration is W_2-flat.

4. Generalized Sasakian-space-forms satisfying $W_2 \cdot S = 0$

Let us take a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ with $W_2 \cdot S = 0$. Then we get

$$S(W_2(X,Y)Z,\xi) + S(Z, W_2(X,Y)\xi) = 0. \tag{4.1}$$

Using (2.14) in (4.1), we get

$$S(W_2(X,Y)Z,\xi) - \frac{1}{2n}\{3f_2 + (2n-1)f_3\}\big[\eta(Y)S(X,Z) - \eta(X)S(Y,Z)\big] = 0. \tag{4.2}$$

Setting $X = \xi$ in (4.2) and using (2.12), (2.13) and (2.15), we get

$$\{3f_2 + (2n-1)f_3\}\big[S(Y,Z) - 2n(f_1 - f_3)\eta(Y)\eta(Z)\big] = 0, \tag{4.3}$$

which implies that either $f_3 = \frac{3f_2}{1-2n}$ or

$$S(Y,Z) = 2n(f_1 - f_3)\eta(Y)\eta(Z).$$

Hence we can state the following:

Theorem 4.1. *If a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ satisfies $W_2 \cdot S = 0$, then either $f_3 = \frac{3f_2}{1-2n}$ or the manifold under consideration is a special type of η-Einstein manifold.*

Remark. The above result is not similar to the case of generalized Sasakian-space-form with $P \cdot S = 0$. In [10] it is shown that if a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ satisfies $P \cdot S = 0$ then either $f_1 = f_3$ or the manifold under consideration is Einstein.

5. W_2-semisymmetric generalized Sasakian-space-forms

Let us suppose that the generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ be W_2-semisymmetric. Then we have

$$R(\xi, U) \cdot W_2 = 0, \tag{5.1}$$

which can be written as

$$\begin{aligned} &R(\xi, U)W_2(X, Y)\xi - W_2(R(\xi, U)X, Y)\xi- \\ &- W_2(X, R(\xi, U)Y)\xi - W_2(X, Y)R(\xi, U)\xi = 0. \end{aligned} \tag{5.2}$$

In view of (2.10), (5.2) yields

$$\begin{aligned} &(f_1 - f_3)\big[g(U, W_2(X, Y)\xi)\xi - \eta(W_2(X, Y)\xi)U- \\ &- g(U, X)W_2(\xi, Y)\xi + \eta(X)W_2(U, Y)\xi - g(U, Y)W_2(X, \xi)\xi+ \\ &+ \eta(Y)W_2(X, U)\xi - \eta(U)W_2(X, Y)\xi + W_2(X, Y)U\big] = 0. \end{aligned} \tag{5.3}$$

Using (2.14) and (2.15) in (5.3), we get

$$\begin{aligned} &(f_1 - f_3)\Bigg[W_2(X, Y)U- \\ &- \frac{1}{2n}\{3f_2 + (2n-1)f_3\}\{g(U, X)Y - g(U, Y)X\}\Bigg] = 0. \end{aligned} \tag{5.4}$$

By virtue of (1.4) we have from (5.4) that

$$\begin{aligned} &(f_1 - f_3)\Big[R(X, Y)U - \frac{1}{2n}\{g(X, U)QY - g(Y, U)QX\}- \\ &- \frac{1}{2n}\{3f_2 + (2n-1)f_3\}\{g(U, X)Y - g(U, Y)X\}\Big] = 0, \end{aligned}$$

which implies that either $f_1 = f_3$ or

$$R(X,Y)U = \frac{1}{2n}\{3f_2 + (2n-1)f_3\}\{g(U,X)Y - g(U,Y)X\} + \frac{1}{2n}\{g(X,U)QY - g(Y,U)QX\}. \tag{5.5}$$

This leads to the following:

Theorem 5.1. *If a generalized Sasakian-space-form is W_2-semisymmetric, then either $f_1 = f_3$ or the curvature tensor of the manifold satisfies the relation* (5.5).

6. Generalized Sasakian-space-forms satisfying $W_2 \cdot R = 0$

We now consider a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ satisfying $W_2 \cdot R = 0$. Then we have

$$W_2(\xi,U)R(X,Y)Z - R(W_2(\xi,U)X,Y)Z - R(X,W_2(\xi,U)Y)Z - R(X,Y)W_2(\xi,U)Z = 0. \tag{6.1}$$

Setting $Z = \xi$ in (6.1) and using (2.9) and (2.15), we get
$\{3f_2 + (2n-1)f_3\}[R(X,Y)U - (f_1 - f_3)\eta(U)\{\eta(Y)X - \eta(X)Y\}] = 0$,
which implies that either $f_3 = \frac{3f_2}{1-2n}$ or

$$R(X,Y)U = (f_1 - f_3)\eta(U)\{\eta(Y)X - \eta(X)Y\}, \quad \text{provided } f_1 \neq f_3. \tag{6.2}$$

This leads to the following:

Theorem 6.1. *If a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ satisfying $W_2 \cdot R = 0$ then either $f_3 = \frac{3f_2}{1-2n}$ or the curvature tensor R of the manifold satisfies the relation* (6.2).

From Th. 3.1 and Th. 7.1, we may conclude the following:

Theorem 6.2. *If a generalized Sasakian-space-form $M^{2n+1}(f_1, f_2, f_3)$ satisfies the condition $W_2 \cdot R = 0$ then either the manifold is W_2-flat or the curvature tensor R of the manifold satisfies the relation* (6.2).

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References

[1] ALEGRE, P., BLAIR, D. E. and CARRIAZO, A.: Generalized Sasakian-space-forms, *Israel J. Math.* **14** (2004), 157–183.

[2] ALEGRE, P. and CARRIAZO, A.: Submanifolds of generalized Sasakian-space-forms, *Taiwanese J. Math.* **13** (2009), 923–941.

[3] ALEGRE, P. and CARRIAZO, A.: Structures on generalized Sasakian-space-forms, *Diff. Geo. and its Application* **26** (2008), 656–666.

[4] ALEGRE, P. and CARRIAZO, A.: Generalized Sasakian-space-forms and conformal changes of the metric, *Results in Math.* **59** (2011), 485–493.

[5] BELKHELFA, M., DESZCZ, R. and VERSTRAELEN, L.: Symmetry properties of generalized Sasakian-space-forms, *Soochow J. Math.* **31** (2005), 611–616.

[6] BLAIR, D. E.: *Contact manifolds in Riemannian geometry*, Lecture Notes in Math. **509**, Springer-Verlag, 1976.

[7] CARRIAZO, A.: On generalized Sasakian-space-forms, in: *Proceedings of the Ninth International Workshop on Diff. Geom.* **9**, 2005, 31–39.

[8] CîRNU, M.: Cohomology and stability of generalized Sasakian-space-forms to appear in *Bull. Malaysian Mathematical Sciences Society.*

[9] DE, U. C. and SARKAR, A.: On a type of P-Sasakian manifolds, *Math. Reports* **11 (61)** (2009), 139–144.

[10] DE, U. C. and SARKAR, A.: On the projective curvature tensor of generalized Sasakian-space-forms, *Quaestiones Mathematicae* **33** (2010), 245–252.

[11] DE, U. C. and SARKAR, A.: Some results on generalized Sasakian-space-forms, *Thai J. Math.* **8** (2010), 1–10.

[12] GHEFARI, R. A., SOLAMY, F. R. A. and SHAHID, M. H.: CR-submanifolds of generalized Sasakian-space-forms, *JP J. Geom. and Topology* **6** (2006), 151–166.

[13] GHERIB, F., GORINE, M. and BELKHELFA, M.: Parallel and semi symmetry of some tensors in generalized Sasakian-space-forms, *Bull. Trans. Univ. Brasov, Series III: Mathematics, Informatics, Physics* **1 (50)** (2008), 139–148.

[14] KIM, U. K.: Conformally flat generalized Sasakian-space-forms and locally symmetric generalized Sasakian-space-forms, *Note di Matematica* **26** (2006), 55–67.

[15] MATSUMOTO, K., IANUS, S. and MIHAI, I.: On P-Sasakian manifolds which admit certain tensor fields, *Publ. Math. Debrecen* **33** (1986), 61–65.

[16] NARAIN, D., YADAV, S. and DWIVEDI, P. K.: On generalized Sasakian-space-forms satisfying certain conditions, *Int. J. Math. and Analysis* **3** (2011), 1–12.

[17] OLTEANU, A.: Legendrian warped product submanifolds in generalized Sasakian-space-forms, *Acta Mathematica Academiae Paedagogice Nyiregyhaziensis* **25** (2009), 137–144.

[18] OLTEANU, A.: A general inequality for doubly warped product submanifolds, *Math. J. Okayama Univ.* **52** (2010), 133–142.

[19] POKHARIYAL, G. P.: Curvature tensors and their relativistic significance III, *Yokohama Math. J.* **20** (1972), 115–119.

[20] POKHARIYAL, G. P.: Study of a new curvature tensor in a Sasakian manifold, *Tensor N. S.* **36** (1982), 222–225.

[21] POKHARIYAL, G. P.: Relative significance of curvature tensors, *Int. J. Math. and Math. Sci.* **5** (1982), 133–139.

[22] POKHARIYAL, G. P.: Curvature tensors on A-Einstein Sasakian manifolds, *Balkan J. Geom. Appl.* **6** (2001), 45–50.

[23] POKHARIYAL, G. P. and MISHRA, R. S.: The curvature tensor and their relativistic significance, *Yokohoma Math. J.* **18** (1970), 105–108.

[24] POKHARIYAL, G. P. and MISHRA, R. S.: Curvature tensor and their relativistic significance II, *Yokohoma Math. J.* **19** (1971), 97–103.

[25] SHAIKH, A. A., JANA, S. K. and EYASMIN, S.: On weakly W_2-symmetric manifolds, *Sarajevo J. Math.* **3 (15)** (2007), 73–91.

[26] SHAIKH, A. A., MATSUYAMA, Y. and JANA, S. K.: On a type of general relativistic spacetime with W_2-curvature tensor, *Indian J. Math.* **50** (2008), 53–62.

[27] SHUKLA, S. S. and CHAUBEY, P. K.: On invariant submanifolds in generalized Sasakian-space-forms, *J. Dynamical Systems and Geometric Theories* **8** (2010), 173–188.

[28] SULAR, S. and ÖZGÜR, C.: Contact CR-warped product submanifolds in generalized Sasakian-space-forms, *Turk. J. Math.* **35** (2011), 1–13.

[29] TALESHIAN, A. and HOSSEINZADEH, A. A.: On W_2-curvature tensor $N(k)$-quasi Einstein manifolds, *J. Math. and Computer Science* **1 (1)** (2010), 28–32.

[30] TRIPATHI, M. M. and GUPTA, P.: On τ-curvature tensor in K-contact and Sasakian manifolds, *Int. Elec. J. Geom.* **4** (2011), 32–47.

[31] VENKATESHA, BAGEWADI, C. S. and KUMAR, K. T. PRADEEP: Some results on Lorentzian Para-Sasakian manifolds, *International scholarly research network* doi:10.5402/2011/161523.

[32] YADAV, S., SUTHAR, D. L. and SRIVASTAVA, A. K.: Some results on $M(f_1, f_2, f_3)_{2n+1}$-manifolds, *Int. J. Pure and Appl. Math.* **70** (2011), 415–423.

[33] YILDIZ, A. and DE, U. C.: On a type of Kenmotsu manifolds, *Diff. Geom.-Dynamical Systems* **12** (2010), 289–298.