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GENERALIZED Λ-SETS AND λ-SETS IN BITOPOLOGICAL SPACES

S. M. **Al-Areefi**

Department of Mathematics, Faculty of Science, P. O. Box 838, Dammam 31113, Kingdom of Saudi Arabia

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Abstract: The aim of this paper is to introduce the concept of generalized Λ-sets in bitopological spaces and define the associated closure operator. Also, we shall introduce the concept of λ-sets in bitopological spaces and establish a new decomposition of pairwise continuity.

1. Introduction

Maki [4] introduced the concept of generalized Λ-sets in topological spaces and defined the associated closure operator C^Λ. G. A. Francisco et al. [1] introduced the concept of λ-sets in topological spaces. T. Fukutake [2] introduced the concept of generalized closed sets in bitopological spaces and a new operator (τ_i, τ_j)-cl*.

The aim of this paper is to introduce the concept of generalized Λ-sets and λ-sets in bitopological spaces and define the associated closure operator ij-cl$^\Lambda$ on the space as an analogy of [2] by generalizing the results in [1, 4] to bitopological spaces.

Throughout this paper, (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or briefly, X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. By i-cl(A) we

E-mail address: dr_sara_m@hotmail.com

denote the closure of A with respect to τ_i (or σ_i) and $X \setminus A = A^C$ will denote the complement of A. Also, $i, j = 1, 2$ and $i \neq j$.

2. ij-$g.\Lambda$-sets

Definition 2.1. Let B be a subset of a bitopological space (X, τ_1, τ_2). We define $B^{\Lambda_i} = \cap\{U : B \subset U,\ U \in \tau_i\}$, $B^{V_i} = \cup\{F : F \subset B,\ X \setminus F \in \tau_i\}$.

Remark 2.2. Let A, B and B_k be subsets of (X, τ_1, τ_2) for every $k \in I$ (an index set), then we have

(1) $B \subset B^{\Lambda_i}$.
(2) If $A \subset B$, then $A^{\Lambda_i} \subset B^{\Lambda_i}$.
(3) $(B^{\Lambda_i})^{\Lambda_i} = B^{\Lambda_i}$.
(4) $\left(\bigcup_{k \in I} B_k\right)^{\Lambda_i} = \bigcup_{k \in I} B_k^{\Lambda_i}$.
(5) If $B \in \tau_i$, then $B = B^{\Lambda_i}$.
(6) $(X \setminus B)^{\Lambda_i} = X \setminus B^{V_i}$.

The converse of (5) above is not true, in general; however, since the intersection of a finite number of τ_i-open sets is τ_i-open, then in finite spaces the converse of (5) is true.

Remark 2.3. We have the following formulas $\left(\bigcap_{k \in I} B_k\right)^{\Lambda_i} \subset \bigcap_{k \in I} B_k^{\Lambda_i}$ and $\left(\bigcup_{k \in I} B\right)^{V_i} \supset \bigcup_{k \in I} B_k^{V_i}$. However, in general, $(B_1 \cap B_2)^{\Lambda_i} \neq B_1^{\Lambda_i} \cap B_2^{\Lambda_i}$ as the following example shows.

Example 2.4. Let $X = \{a, b, c\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}\}$ and $\tau_2 = = \{X, \emptyset, \{b\}\}$. Let $B_1 = \{b\}$ and $B_2 = \{c\}$. Then $(B_1 \cap B_2)^{\Lambda_1} = \emptyset$ but $B_1^{\Lambda_1} \cap B_2^{\Lambda_1} = \{b, c\}$.

Definition 2.5. A subset B of a bitopological space (X, τ_1, τ_2) is called Λ_i-set (resp. V_i-set) if $B = B^{\Lambda_i}$ (resp. $B = B^{V_i}$). Then we have the following properties.

(1) $\emptyset$ and X are Λ_i-sets.
(2) Every union of Λ_i-sets is a Λ_i-set.
(3) Every intersection of Λ_i-sets is a Λ_i-set.
(4) A subset B is Λ_i-set if and only if $X \setminus B$ is a V_i-set.

Thus we conclude that the family of all Λ_i-sets of X forms a topology on X finer than τ_i.

To show that not every Λ_i-set is τ_i-open and so the converse of (5) in Rem. 2.2 is not true, in general, let $X = \mathbb{R}$ be the set of all real numbers, $\tau_1 =$ the usual topology and $\tau_2 =$ the indiscrete topology on $\mathbb{R}$. Then $\{0\}$ is a Λ_1-set but not τ_1-open.

It is easy to see that in a bitopological space (X, τ_1, τ_2), if every singleton set is a Λ_i-set, then (X, τ_1, τ_2) is pairwise T_1 in the sense of Reilly [5].

Proposition 2.6. *If a bitopological space (X, τ_1, τ_2) is pairwise T_1 in the sense of Reilly* [5]*, then every subset of X is a Λ_i-set.*

Proof. Let B be a subset of X and $x \in X \setminus B$. Since X is pairwise T_1, therefore $\{x\}$ is τ_i-closed and so $X \setminus \{x\}$ is a τ_i-open set containing B. This implies that $x \notin B^{\Lambda_i}$. Hence we have $B^{\Lambda_i} \subset B$ and therefore $B^{\Lambda_i} = B$. ◊

Definition 2.7. A subset B of a bitopological space (X, τ_1, τ_2) is called an ij-$g.\Lambda$-set if $B^{\Lambda_i} \subset F$ whenever $B \subset F$ and F is τ_j-closed. A subset B is called an ij-$g.V$-set of X if $X \setminus B$ is ij-$g.\Lambda$-set. Let ij-D^{Λ} (resp. ij-D^{V}) denote the set of all ij-$g.\Lambda$-sets (resp. ij-$g.V$-sets) of X.

Remark 2.8. In a bitopological space (X, τ_1, τ_2), we have

(1) Every Λ_i-set is an ij-$g.\Lambda$-set.

(2) Every V_i-set is an ij-$g.V$-set.

(3) If $B_k \in ij\text{-}D^{\Lambda}$, then $\bigcup_{k \in I} B_k \in ij\text{-}D^{\Lambda}$ for any index set I.

(4) If $B_k \in ij\text{-}D^{V}$, then $\bigcap_{k \in I} B_k \in ij\text{-}D^{V}$ for any index set I.

The intersection of two ij-$g.\Lambda$-sets need not be an ij-$g.\Lambda$-set as shown by the following example.

Example 2.9. Let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $\tau_2 = \{X, \emptyset, \{b\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, b, c\}$ and $\{a, b, d\}$ are 21-$g.\Lambda$-sets but their intersection $\{a, b\}$ is not a 21-$g.\Lambda$-set.

The converse of Rem. 2.8 (1) is not true, in general, since in Ex. 2.9, $\{b, c\}$ and $\{a, b, d\}$ are 12-$g.\Lambda$-sets but not Λ_1-sets.

Proposition 2.10. *Let x be a point of a bitopological space (X, τ_1, τ_2), then*

(1) *$\{x\}$ is a τ_j-open set or $X \setminus \{x\}$ is an ij-$g.\Lambda$-set.*

(2) *$\{x\}$ is a τ_j-open set or $\{x\}$ is an ij-$g.V$-set.*

Proof. (i) Suppose that $\{x\}$ is not a τ_j-open set. Then the only τ_j-closed set F containing $X \setminus \{x\}$ is X. Thus $(X \setminus \{x\})^{\Lambda_i} \subset F = X$ and $X \setminus \{x\}$ is an ij-$g.\Lambda$-set.

(ii) Follows directly by (i) and the definition. ◊

Remark 2.11. In a bitopological space (X, τ_1, τ_2)

(i) for a subset $B \subset X$, generally, B^{Λ_1} is not equal to B^{Λ_2}, since in Ex. 2.9, $\{a, c\}^{\Lambda_1} = \{a, c, d\}$ but $\{a, c\}^{\Lambda_2} = X$;

(ii) generally, 12-D^{Λ} is not equal to 21-D^{Λ}, since in Ex. 2.9, 12-$D^{\Lambda} =$ $=$ the set of all subsets of X while 21-$D^{\Lambda} = \{\{b\}, \{c\}, \{d\}, \{c, d\}, \{b, c\}, \{b, d\}, \{b, c, d\}, \{a, b, c\}, \{a, b, d\}, X, \emptyset\}$.

Proposition 2.12. *In a bitopological space (X, τ_1, τ_2), if $\tau_i \subset \tau_j$, then $B^{\Lambda_j} \subset B^{\Lambda_i}$ and ij-$D^{\Lambda} \subset ji$-D^{Λ}, for any subset B of X.*

Proposition 2.13. *In a bitopological space (X, τ_1, τ_2), every subset A is an ij-$g.\Lambda$-set if and only if every τ_j-closed set is a Λ_i-set.*

Proof. Let $A \subset X$ and let F be a τ_j-closed set such that $A \subset F$. Then $A^{\Lambda_i} \subset F^{\Lambda_i} = F$ (since F is a Λ_i-set). This shows that A is an ij-$g.\Lambda$-set. Conversely, let F be a τ_j-closed set, then F is an ij-$g.\Lambda$-set and so $F^{\Lambda_i} \subset F$, but $F \subset F^{\Lambda_i}$, then $F = F^{\Lambda_i}$. This shows that F is a Λ_i-set. $\Diamond$

Proposition 2.14. *In a bitopological space (X, τ_1, τ_2), if B is an ij-$g.\Lambda$-set, then $B^{\Lambda_i} \setminus B$ contains no nonempty τ_j-open set.*

Proof. Let U be a τ_j-open subset of $B^{\Lambda_i} \setminus B$, then $B \subset U^C$. Now, since U^C is τ_j-closed and B is an ij-$g.\Lambda$-set, then $B^{\Lambda_i} \subset U^c$, i.e., $U \subset (B^{\Lambda_i})^C$. Thus $U \subset B^{\Lambda_i} \cap (B^{\Lambda_i})^C = \emptyset$. $\Diamond$

Proposition 2.15. *In a bitopological space (X, τ_1, τ_2), if A is an ij-$j.\Lambda$-set and $A \subset B \subset A^{\Lambda_i}$, then B is an ij-$g.\Lambda$-set.*

3. ij-Λ-closure operator

By using the family of all ij-$g.\Lambda$-sets of a bitopological space (X, τ_1, τ_2), we define the closure operator ij-cl^{Λ} and the associated topology $\tau^{\Lambda}(i, j)$ on X.

Definition 3.1. For a subset B of a bitopological space (X, τ_1, τ_2), we define ij-$\mathrm{cl}^{\Lambda}(B) = \cap\{U : B \subset U, U \in ij\text{-}D^{\Lambda}\}$ and ij-$\mathrm{int}^{\Lambda}(B) = \cup\{F : F \subset$ $\subset B, F \in ij\text{-}D^{V}\}$.

Proposition 3.2. *Let A, B and B_k be subsets of a bitopological space (X, τ_1, τ_2) for $k \in I$, then*

(1) $B \subset ij$-$\mathrm{cl}^{\Lambda}(B)$, ij-$\mathrm{cl}^{\Lambda}(X \setminus B) = X \setminus ij$-$\mathrm{int}^{\Lambda}(B)$ *and* ij-$\mathrm{cl}^{\Lambda}(\emptyset) = \emptyset$.

(2) $\bigcup_{k \in I} ij\text{-}\mathrm{cl}^{\Lambda}(B_k) = ij\text{-}\mathrm{cl}^{\Lambda}\Big(\bigcup_{k \in I} B_k\Big)$.

(3) ij-$\mathrm{cl}^{\Lambda}(ij$-$\mathrm{cl}^{\Lambda}(B)) = ij$-$\mathrm{cl}^{\Lambda}(B)$.

(4) *If $A \subset B$, then ij-$\mathrm{cl}^{\Lambda}(A) \subset ij$-$\mathrm{cl}^{\Lambda}(B)$.*

Proof. The proofs of (1) and (4) are obvious.

(2) Suppose that there exists a point $x \notin ij\text{-cl}^\Lambda\Big(\bigcup_{k\in I} B_k\Big)$. Then there exists a subset $U \in ij\text{-}D^\Lambda$ such that $\bigcup_{k\in I} B_k \subset U$ and $x \notin u$. Thus for each $k \in I$, we have $x \notin \bigcup_{k\in I} ij\text{-cl}^\Lambda(B_k)$. Conversely, let $x \notin \bigcup_{k\in I} ij\text{-cl}^\Lambda(B_k)$. Then there exist subsets $U_k \in ij\text{-}D^\Lambda$, for all $k \in I$, such that $x \notin U_k$ and $B_k \subset U_k$. Let $U = \bigcup_{k\in I} U_k$, then $x \notin U$, $U \in ij\text{-}D^\Lambda$ and $\bigcup_{k\in I} B_k \subset U$. This shows that $x \notin ij\text{-cl}^\Lambda\Big(\bigcup_{k\in I} B_k\Big)$.

(3) It is clear by the definition that $ij\text{-cl}^\Lambda(B) \subset ij\text{-cl}^\Lambda(ij\text{-cl}^\Lambda(B))$. Now let $x \notin ij\text{-cl}^\Lambda(B)$, then there exists $U \in ij\text{-}D^\Lambda$ such that $x \notin U$ and $B \subset U$. Since $U \in ij\text{-}D^\Lambda$, then $ij\text{-cl}^\Lambda(B) \subset U$, thus $x \notin ij\text{-cl}^\Lambda(ij\text{-cl}^\Lambda(B))$. This shows that $ij\text{-cl}^\Lambda(ij\text{-cl}^\Lambda(B)) \subset ij\text{-cl}^\Lambda(B)$. $\Diamond$

By Prop. 3.2, we have the following theorem.

Theorem 3.3. *The operator ij-cl$^\Lambda$ defined above is a Kuratowski operator on X.*

From Th. 3.3, ij-cl$^\Lambda$ defines a new topology on X.

Definition 3.4. Let $\tau^\Lambda(i,j)$ be the topology on X generated by ij-cl$^\Lambda$ in the usual manner, i.e., $\tau^\Lambda(i,j) = \{E \subset X : ij\text{-cl}^\Lambda(X \setminus E) = X \setminus E\}$. We define a family $ij\text{-}F^\Lambda$ by $ij\text{-}F^\Lambda = \{B : ij\text{-cl}^\Lambda(B) = B\}$. Then $ij\text{-}F^\Lambda = \{B : X \setminus B \in \tau^\Lambda(i,j)\}$. And let $\tau^V(i,j) = \{B \subset X : ij\text{-int}^V(B) = B\}$.

Proposition 3.5. *Let F_i^Λ (resp. τ_i^V) be the family of all Λ_i-sets (resp. V_i-sets) of (X, τ_1, τ_2). Then*

(1) $\tau^\Lambda(i,j) = \tau^V(i,j)$.

(2) $\tau_i \subset F_i^\Lambda \subset ij\text{-}D^\Lambda \subset ij\text{-}F^\Lambda$.

(3) $F_i \subset \tau_i^V \subset ij\text{-}D^V \subset \tau^\Lambda(i,j)$, *where F_i is the family of all τ_i-closed subsets of X.*

Proof. (1) It is clear by Rem. 2.2 (6) and Def. 3.4. To prove (2), let $B \in \tau_i$, then, by Rem. 2.2 (5), B is a Λ_i-set, i.e., $B \in F_i^\Lambda$. By Rem. 2.8 (1), $B \in ij\text{-}D^\Lambda$ for each $B \in F_i^\Lambda$. For each $B \in ij\text{-}D^\Lambda$, we have $B = ij\text{-cl}^\Lambda(B)$. This implies that $B \in ij\text{-}F^\Lambda$. The proof of (3) follows directly by Rem. 2.2 (5), (6), Rem. 2.8 (2) and Def. 3.4. $\Diamond$

Remark 3.6. Containment relations in Prop. 3.5 may be proper, since in Ex. 2.9, $\{a,b\} \in 21\text{-}F^\Lambda$ but $\{a,b\} \notin 21\text{-}D^\Lambda$ and $\{a,b,c\} \in 21\text{-}D^\Lambda$ but $\{a,b,c\}$ is not a Λ_2-set.

Remark 3.7. Let (X, τ_1, τ_2) be as in Ex. 2.9. Then $21\text{-cl}^\Lambda(\{a,d\}) =$

$= \{a, b, d\}$ and (τ_2, τ_1)-cl$^*(\{a, d\}) = \{a, d\}$, where (τ_i, τ_j)-cl* is the closure operator due to T. Fukutake [2]. In this space 21-$D^\Lambda = \{X, \emptyset, \{b\}, \{c\}, \{d\}, \{c, d\}, \{b, c\}, \{b, d\}, \{b, c, d\}, \{a, b, c\}, \{a, b, d\}\}$ and $\tau^\Lambda(2, 1) = \{X, \emptyset, \{a\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$. On the other hand, $D(\tau_2, \tau_1) =$ the power of X and $\tau^*(\tau_2, \tau_1) =$ the indiscrete topology on X, where $D(\tau_i, \tau_j)$ is the family of all (τ_i, τ_j)-g-closed sets (A is called (τ_i, τ_j)-g-closed if and only if j-cl$(A) \subset G$ whenever $A \subset$ $\subset G$ and G is τ_i-open [2]) and $\tau^*(\tau_i, \tau_j)$ is the topology on X generated by (τ_i, τ_j)-cl* operator [2]. These examples show that (τ_i, τ_j)-cl* operator does not coincide with ij-cl$^\Lambda$ operator and $\tau^*(\tau_i, \tau_j)$ and $\tau^\Lambda(i, j)$ are different topologies on X, in general.

Proposition 3.8. *In a bitopological space (X, τ_1, τ_2), if U is an ij-g-Λ-set, then ij-cl$^\Lambda(U) = U$.*

Corollary 3.9. *In a bitopological space (X, τ_1, τ_2), if E is an ij-$g.\Lambda$-set, then E is $\tau^\Lambda(i, j)$-closed.*

Proposition 3.10. *For a bitopological space (X, τ_1, τ_2), we have*

(1) *If $F_i = \tau^\Lambda(i, j)$, then $\tau_i = ij$-D^Λ, where F_i is the family of all τ_i-closed sets of X.*

(2) *If every ij-$g.\Lambda$-set is τ_i-open, then $\tau_i^V = \tau^\Lambda(i, j)$.*

(3) *If every ij-$g.\Lambda$-closed set is τ_i-closed, then $\tau_i = \tau^\Lambda(i, j)$.*

Proof. (1) By Prop. 3.5 (2), $\tau_i \subset ij$-D^Λ. Now, let B be any ij-$g.\Lambda$-set, i.e., $B \in ij$-D^Λ. Then by Prop. 3.5 (2), $B \in ij$-F^Λ. This implies that $X \setminus B \in \tau^\Lambda(i, j)$. From this assumption, we have $X \setminus B \in F_i$ and so $B \in \tau_i$. This shows that ij-$D^\Lambda \subset \tau_i$.

(2) By Prop. 3.5 (3), $\tau_i^V \subset \tau^\Lambda(i, j)$. Now, let $B \in \tau^\Lambda(i, j)$, then by the definition $X \setminus B = ij$-cl$^\Lambda(X \setminus B) = \cap\{U : X \setminus B \subset U, U \in ij$-$D^\Lambda\} =$ $= \cap\{U : X \setminus B \subset U, U \in \tau_i\} = (X \setminus B)^{\Lambda_i}$. Hence $B = B^{V_i}$ and $B \in \tau_i^V$. This shows that $\tau^\Lambda(i, j) \subset \tau_i^V$.

(3) The proof is similar to that of (2). $\Diamond$

Proposition 3.11. *In a bitopological space (X, τ_1, τ_2), if $\tau_j = \tau^\Lambda(i, j)$, then every singleton $\{x\}$ of X is $\tau^\Lambda(i, j)$-open.*

Proof. Suppose that $\{x\}$ is not τ_j-open, then by Prop. 2.10 (1), $X \setminus \{x\}$ is ij-$g.\Lambda$-set. Thus $\{x\} \in \tau^\Lambda(i, j)$. Now if $\{x\} \in \tau_j = \tau^\Lambda(i, j)$, then $\{x\}$ is $\tau^\Lambda(i, j)$-open. $\Diamond$

4. *ij*-λ-sets

Definition 4.1. A subset A of a bitopological space (X, τ_1, τ_2) is called ij-λ-closed if $A = L \cap F$, where L is a Λ_i-set and F is a τ_j-closed set. The complement of an ij-λ-closed set will be called ij-λ-open.

Remark 4.2. The concepts of ij-$g.\Lambda$-sets and ij-λ-closed sets are independent, since let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $\tau_2 = \{X, \emptyset, \{b\}, \{c, d\}, \{b, c, d\}\}$. Then $\{a, b, d\}$ and $\{b, c\}$ are both 21-$g.\Lambda$-sets but not 21-λ-closed. Also, $\{a, c, d\}$ and $\{a\}$ are both 21-λ-closed sets but not 21-$g.\Lambda$-sets.

Proposition 4.3. *For a subset A of a bitopological space (X, τ_1, τ_2), the following are equivalent:*

(1) *A is ij-λ-closed.*

(2) *$A = L \cap j\text{-cl}(A)$, where L is a Λ_i-set.*

(3) *$A = A^{\Lambda_i} \cap j\text{-cl}(A)$.*

Proposition 4.4. *In a bitopological space (X, τ_1, τ_2),*

(1) *Every ij-locally closed set is ij-λ-closed (A is ij-locally closed if $A = U \cap F$, where U is a τ_i-open set and F is τ_j-closed* [3]*).*

(2) *Every Λ_i-set is ij-λ-closed.*

Generally, ij-locally closet sets and Λ_i-sets are independent concepts, and so an ij-closed set need not be an ij-locally closed or a Λ_i-set. However, in finite spaces, the concept of ij-locally closed sets coincides with the concept of ij-λ-closed sets.

Definition 4.5 ([2]). A subset A of a bitopological space (X, τ_1, τ_2) is said to be (τ_1, τ_2)-g-closed if $A \subset G \in \tau_1$ implies $\tau_2\text{-cl}\, A \subset G$.

Proposition 4.6. *A subset A of a bitopological space (X, τ_1, τ_2) is (τ_i, τ_j)-g-closed if and only if $j\text{-cl}(A) \subset A^{\Lambda_i}$.*

Theorem 4.7. *For a subset A of a bitopological space (X, τ_1, τ_2), the following are equivalent:*

(1) *A is τ_j-closed.*

(2) *A is (τ_i, τ_j)-g-closed and ij-locally closed.*

(3) *A is (τ_i, τ_j)-g-closed and ij-λ-closed.*

Proof. (1)⇒(2): Every τ_j-closed set is both (τ_i, τ_j)-g-closed and ij-locally closed.

(2)⇒(3): That is Prop. 4.4 (1).

(3)⇒(1): A is (τ_i, τ_j)-g-closed, so by Prop. 4.6, $j\text{-cl}(A) \subset A^{\Lambda_i}$. A is ij-λ-closed so by Prop. 4.3, $A = A^{\Lambda_i} \cap j\text{-cl}(A)$. Hence $A = j\text{-cl}(A)$ and A is j-closed. ◊

Theorem 4.8. *A bitopological space (X, τ_1, τ_2) is pairwise T_0 if and only if every singleton subset of X is ij-λ-closed.*

Proof. Let $x \in X$. Since X is pairwise T_0, therefore, for every point $y \neq x$, there exists a set A_y containing x and disjoint from $\{y\}$ such that A_y is either τ_i-open or τ_j-closed. Let L be the intersection of all τ_i-open sets A_y and let F be the intersection of all τ_j-closed sets A_y. Clearly, L is a Λ_i-set and F is τ_j-closed. Note that $\{x\} = L \cap F$. This shows that $\{x\}$ is ij-λ-closed. Conversely, let x and y be two different points of X. Then $\{x\} = L \cap F$, where L is a Λ_i-set and F is τ_j-closed. If F does not contain y, then $X \setminus F$ is a τ_j-open set containing y and we are done. If F contains y, then $y \notin L$ and thus for some τ_i-open set U containing x, we have $y \notin U$. Hence X is pairwise T_0. $\Diamond$

Definition 4.9 ([2])**.** A bitopological space (X, τ_1, τ_2) is called ij-$T_{1/2}$ if any (τ_i, τ_j)-g-closed set is τ_j-closed. (X, τ_1, τ_2) is called strongly pairwise $T_{1/2}$ if it is 12-$T_{1/2}$ and 21-$T_{1/2}$.

Theorem 4.10 ([2])**.** *A bitopological space (X, τ_1, τ_2) is ij-$T_{1/2}$ if and only if $\{x\}$ is τ_j-open or τ_j-closed for each $x \in X$.*

Theorem 4.11. *A bitopological space (X, τ_1, τ_2) is ij-$T_{1/2}$ if and only if every subset of X is ij-λ-closed.*

Proof. Let $A \subset X$, A_1 be the set of all τ_j-open singleton of $X \setminus A$ and let $A_2 = X \setminus (A \cup A_1)$. Set $F = \bigcap_{x \in A_1} X \setminus \{x\}$ and $L = \bigcap_{x \in A_2} X \setminus \{x\}$. Note that F is τ_j-closed and L is a Λ_i-set. Moreover, $A = F \cap L$. Thus A is ij-λ-closed. Conversely, let $x \in X$. If $\{x\}$ is not τ_j-open, then $A = X \setminus \setminus \{x\}$ is not τ_j-closed. Since A is ij-λ-closed, therefore A is a Λ_i-set, i.e., $A = A^{\Lambda_i}$. Since X is the only superset of A, therefore A is τ_i-open and $\{x\}$ is τ_i-closed. $\Diamond$

Definition 4.12. A bitopological space (X, τ_1, τ_2) is called pairwise $T_{1/4}$ if for every finite subset F of X and every $y \notin F$, there exists a set A_y containing F and disjoint from $\{y\}$ such that A_y is either τ_i-open or τ_j-closed.

Theorem 4.13. *A bitopological space (X, τ_1, τ_2) is pairwise $T_{1/4}$ if and only if every finite subset of X is ij-λ-closed.*

Proof. Let $F \subset X$ be a finite subset. Since X is pairwise $T_{1/4}$, for every point $y \notin F$, there exists a set A_y containing F and disjoint from $\{y\}$ such that A_y is τ_i-open or τ_j-closed. Let L be the intersection of all τ_i-open sets A_y and C be the intersection of all τ_j-closed sets A_y. Clearly, L is a Λ_i-set and C is τ_j-closed. Note that $F = L \cap C$. This shows that

F is ij-λ-closed. Conversely, let F be a finite subset of X and $y \in X \setminus F$. Then $F = L \cap C$ where L is a Λ_i-set and C is τ_j-closed. If C does not contain y, then $X \setminus C$ is a τ_j-open set containing y. If C contains y, then $y \notin L$ and thus for some τ_i-open set U containing F, we have $y \notin U$. Hence X is pairwise $T_{1/4}$. $\Diamond$

A finite union of ij-λ-closed sets need not be ij-λ-closed. However, since any intersection of Λ_i-sets is a Λ_i-set, we have the following:

Proposition 4.14. *In a bitopological space (X, τ_1, τ_2), an arbitrary intersection of ij-λ-closed sets is ij-λ-closed.*

Now, one can ask the following question: for which spaces, the set of all ij-λ-open subsets is a topology? Call those spaces ij-λ-spaces.

Clearly, a bitopological space (X, τ_1, τ_2) is an ij-λ-space if and only if the union of two ij-λ-closed sets is ij-λ-closed. From Th. 4.11, we have that every ij-$T_{1/2}$ space is an ij-λ-space. Also, a pairwise T_0 ij-λ-space is pairwise $T_{1/4}$, since from Th. 4.8, every singleton is ij-λ-closed and in an ij-λ-space, finite union of ij-λ-closed sets is ij-λ-closed.

Definition 4.15. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(1) ij-g.continuous if the inverse image of each σ_j-closed set in Y is (τ_i, τ_j)-g.closed in X. f is called pairwise g-continuous if it is 12-g-continuous and 21-g-continuous.

(2) ij-co-LC-continuous if $f^{-1}(V)$ is ij-locally closed in X for every σ_j-closed V of Y. f is called pairwise co-LC-continuous if it is 12-co-LC-continuous and 21-co-LC-continuous.

(3) ij-λ-continuous if $f^{-1}(V)$ is ij-λ-closed in X for every σ_j-closed V of Y. f is called pairwise λ-continuous if it is 12-λ-continuous and 21-λ-continuous.

Every ij-co-LC-continuous function is ij-λ-continuous but the converse mayx not be true, in general, as can be shown by the following example:

Example 4.16. Let $\mathbb{R}$ be the set of all real numbers, τ_1 = the usual topology on $\mathbb{R}$, τ_2 = the indiscrete topology on $\mathbb{R}$, σ_1 = the cofinite topology on $\mathbb{R}$ and $\sigma_2 = \{\mathbb{R}, \emptyset, \mathbb{R} \setminus \{0\}\}$. The identity function $f : (\mathbb{R}, \tau_1, \tau_2) \to$ $\to (\mathbb{R}, \sigma_2, \sigma_2)$ is 12-λ-continuous but not 12-co-LC-continuous. Since $\{0\}$ is the only proper σ_2-closed set and $f^{-1}(\{0\}) = \{0\}$ is 12-λ-closed because $\{0\} = \{0\} \cap \mathbb{R}$, $\{0\}$ is a Λ_1-set and $\mathbb{R}$ is τ_2-closed. But $\{0\}$ is not 12-locally closed. Indeed, $\{0\}$ is neither τ_1-open nor τ_2-closed.

To see that ij-g-continuity and ij-λ-continuity are concepts totally independent of each other, consider the following two examples:

Example 4.17. Let $X = \{a, b, c, d\}$, $\tau_1 = \{X, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $\tau_2 = \{X, \emptyset, \{a, b\}, \{c, d\}\}$. Let $Y = X$, $\sigma_1 = \{Y, \emptyset, \{b\}, \{d\}, \{b, d\}\}$ and $\sigma_2 = \{Y, \emptyset, \{a\}, \{c\}, \{a, c\}, \{b, c, d\}\}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be the identity function. Then f is 21-g-continuous but not 21-λ-continuous.
Example 4.18. Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be as in Ex. 4.17 and let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = f(b) = a$, $f(c) = f(d) = $ $= d$. Then f is 21-λ-continuous but not 21-g-continuous.

Finally, we present a new decomposition of j-continuity and pairwise continuity.
Theorem 4.19. *For a function $f : X \to Y$, the following are equivalent:*
(1) *f is j-continuous.*
(2) *f is ij-g-continuous and ij-co-LC-continuous.*
(3) *f is ij-g-continuous and ij-λ-continuous.*
Corollary 4.20. *For a function $f : X \to Y$, the following are equivalent:*
(1) *f is pairwise continuous.*
(2) *f is pairwise g-continuous and pairwise co-LC-continuous.*
(3) *f is pairwise g-continuous and pairwise λ-continuous.*

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