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SEMI-RADIALITY IN PRODUCTS

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Abstract: We loo at the product of semi-radial spaces. Slightly improving a result of D. V. Malykhin, we show that the product of countably many compact semi-radial spaces is almost radial. Further, we consider conditions under which a product is semi-radial.

Considerations around the product continue to be a central theme in the study of pseudoradial spaces, see for instance [1], [2] and [3].

In 1997 D. V. Malykhin [5] proved that the product of countably many compact semi-radial spaces is pseudoradial and such result has been recently improved by E. Murtinová [6] by showing that a product of ω_1 compact semi-radial spaces is pseudoradial if it is sequentially compact.

Here we present some more thinks on the product of semi-radial spaces, We begin by proving that the product of countably many compact semi-radial spaces is actually almost radial (a bit more than Malykhin's result). Then we look at some conditions for the semi-radiality of a product.

For more on the various kinds of pseudoradial spaces see for instance [1].

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By transfinite sequence we mean any well ordered net, i.e. a set indexed by some ordinal.

A transfinite sequence S in the topological space X converges (strictly converges) to $x \in X$ if each neighbourhood of X contains a final segment of S (and moreover x is not in the closure of any initial segment of S).

Given a space X and a set $A \subseteq X$, we denote by A^r (A^{sr}) the smallest subset of X that contains A and is radially closed (strictly radially closed), i.e. no sequence in A^r (A^{sr}) can converge (strictly converge) to a point outside A^r (A^{sr}).

A space is pseudoradial (almost radial) if $\overline{A} = A^r$ ($\overline{A} = A^{sr}$) holds for each $A \subseteq X$.

A subset A of a space X is κ-closed ($< \kappa$- closed) if $\overline{B} \subseteq A$ for any set $B \subseteq A$ satisfying $|B| \leq \kappa$ ($|A| < \kappa$).

A space X is semi-radial provided that every non-κ- closed set $A \subseteq X$ contains a sequence of length at most κ which converges to a point outside A.

A space X is R-monolithic if for any $A \subseteq X$ and any non-closed set $B \subseteq \overline{A}$ there exists a sequence $S \subseteq B$ which converges to a point outside B and satisfies $|S| \leq |A|$.

We have

R-monolithic $\Longrightarrow$ *semi-radial* $\Longrightarrow$ *almost radial* $\Longrightarrow$ *pseudoradial.*

Proposition 1 [7]. *The product of finitely many compact semi-radial spaces is almost radial.*

Theorem 1. *The product of countably many compact semi-radial spaces is almost radial.*

Proof. Let $X = \prod_{n \in \omega} X_n$ and assume that each X_n is a compact semi-radial space. If $F \subseteq \omega$, then $\pi_F : X \to X_F = \prod_{n \in F} X_n$ denotes the projection. When $F = \{n\}$ we simply write π_n.

Arguing by contradiction, we assume that there exists a non-closed strictly radially closed set $A \subseteq X$. Notice first that there is some finite set $F \subseteq \omega$ such that $\pi_F[A]$ is not closed in X_F (otherwise we can easily find a ω-sequence in A converging to some point of $\overline{A} \setminus A$). Since X_F is almost radial, there is a sequence $B \subseteq A$ such that $\pi_F[B]$ strictly converges to some point $x_F \in X_F \setminus \pi_F[A]$. In particular, we have that $x_F \notin \pi_F[B^{sr}]$.

Choose now F and B in such a way that B has the smallest possible length, say $|B| = \kappa$. Clearly, κ is a regular cardinal.

If $\kappa = \omega$, then the sequential compactness of X would provide a convergent subsequence $S \subseteq B$ and this in turn would give $x_F = \lim \pi_F[B] = \lim \pi_F[S] \in \pi_F[S^{sr}] \subseteq \pi_F[B^{sr}]$. Therefore κ is uncountable.

Claim 1: *If $S \subseteq X$ satisfies $|S| < \kappa$, then $\overline{\pi_n[S]} \subseteq \pi_n[S^{sr}]$ for each $n \in \omega$.*

Since $\pi_n[S] \subseteq \pi_n[S^{sr}] \subseteq \overline{\pi_n[S]}$ and X_n is semi-radial, it follows that the formula $\overline{\pi_n[S]} \setminus \pi_n[S^{sr}] \neq \emptyset$ would imply the existence of a sequence $C \subseteq S^{sr}$, of length at most $|S| < \kappa$, whose projection $\pi_n[C]$ is strictly convergent to some point $x_n \notin \pi_n[S^{sr}] \supseteq \pi_n[C^{sr}]$. This is in contradiction with the minimality of κ and so Claim 1 is proved.

Claim 2: *For any $i \in \omega$ there exist a point $x_i \in X_i$ and a sequence $B' \subseteq B^{sr}$ such that $|B'| = \kappa$, $\lim \pi_i[B'] = x_i$ and $\lim \pi_j[B] = x_j \Longrightarrow \lim \pi_j[B'] = x_j$ for every $j \in \omega$.*

Let $B = \{b_\alpha : \alpha < \kappa\}$ and $B_\alpha = \{b_\beta : \beta < \alpha\}$. Then let x_i be a complete accumulation point of the set $\pi_i[B]$.

CASE 1: for any $\alpha < \kappa$ there exists $\beta(\alpha) < \kappa$ such that $x_i \in \overline{\pi_i[B_{\beta(\alpha)} \setminus B_\alpha]}$. Thanks to Claim 1, we can pick $b'_\alpha \in (B_{\beta(\alpha)} \setminus B_\alpha)^{sr} \subseteq B^{sr}$ in such a way that $\pi_i(b'_\alpha) = x_i$. The assertion $\lim \pi_j[B] = x_j$ means that for any closed neighbourhood U of x_j there exists $\gamma < \kappa$ such that $\pi_j[B \setminus B_\gamma] \subseteq U$ or, equivalently, $B \setminus B_\gamma \subseteq \pi_j^{-1}[U]$. Hence, for any $\alpha > \gamma$, we have $b'_\alpha \in (B_{\beta(\alpha)} \setminus B_\alpha)^{sr} \subseteq (B \setminus B_\gamma)^{sr} \subseteq \pi_j^{-1}[U]$ and thus, letting $B' = \{b'_\alpha : \alpha < \kappa\}$, we have $\lim \pi_j[B'] = x_j$. The sequence B' has then all the required properties.

CASE 2: by cutting off some initial segment of B, we may assume that $x_i \notin \overline{\pi_i[B_\alpha]}$ for every $\alpha < \kappa$. Letting $C = \bigcup\{\overline{\pi_i[B_\alpha]} : \alpha < \kappa\}$, we get a non-closed subset of X_i (clearly $x_i \in \overline{\pi_i[B]} = \overline{C}$ and $x_i \notin C$). Since X_i is semi-radial, there exists a sequence $S \subseteq C$, necessarily of length κ, which strictly converges to some point $x'_i \in \overline{C} \setminus C$. Writing $S = \{s_\alpha : \alpha < \kappa\}$, we may assume that $s_\alpha \in \overline{\pi_i[B_{\beta(\alpha)}]} \setminus \overline{\pi_i[B_\alpha]} \subseteq \overline{\pi_i[B_{\beta(\alpha)} \setminus B_\alpha]}$, for some $\beta(\alpha) \in \kappa \setminus \alpha$. Again by Claim 1, we may then pick points $b'_\alpha \in (B_{\beta(\alpha)} \setminus B_\alpha)^{sr} \subseteq B^{sr}$ in such a way that $\pi_i(b'_\alpha) = s_\alpha$. By replacing x_i with x'_i, we see that the sequence $B' = \{b'_\alpha : \alpha < \kappa\}$ has even in this case all the required properties. Claim 2 is then proved.

Now, we come back to the proof of the theorem. Using countably many times Claim 2, we can construct for each $n \in \omega$ a sequence $B_n = \{b^n_\alpha : \alpha < \kappa\}$ such that $B_{n+1} \subseteq B_n^{sr} \subseteq B^{sr}$, $\lim \pi_n[B_n] = x_n$ for some $x_n \in X_n$ and $\lim \pi_j[B_n] = x_j$ for each $j < n$.

It is clear that if $n \in F$ then $x_n = \pi_n(x_F)$, therefore if x is the point of X whose n-th coordinate is x_n then we have $\pi_F(x) = x_F$. For any $\alpha < \kappa$ select a point $d_\alpha \in \{b^n_\alpha : n \in \omega\}^{sr} \subseteq B^{sr}$. The sequence $D = \{d_\alpha : \alpha < \kappa\}$ so obtained converges to x. To check this, it is enough to verify that for any finite set $G \subseteq \omega$ and any closed neighbourhood U of $\pi_G(x)$ in X_G there exists some $\gamma < \kappa$ such that $D \setminus D_\gamma \subseteq \pi_G^{-1}[U]$. Choose n such that $G \subseteq n$ and observe that the formula $\lim \pi_j[B_m] = x_j$ for every $j \leq m$ actually implies that $\lim \pi_G[B_m] = \pi_G(x)$ holds for each $n \leq m$. Therefore, for each $m \geq n$ there exists some $\gamma_m < \kappa$ such that $B_m \setminus (B_m)_{\gamma_m} \subseteq \pi_G^{-1}[U]$. Since κ is regular and uncountable, we may pick some $\gamma < \kappa$ such that $\gamma_m < \gamma$ for each $m \geq n$. Consequently, for every $\alpha \geq \gamma$, we have $\{b^m_\alpha : n \leq m < \omega\} \subseteq \pi_G^{-1}[U]$ and this in turn gives $d_\alpha \in \{b^m_\alpha : n \leq m < \omega\}^{sr} \subseteq \pi_G^{-1}[U]$. Hence, $D \setminus D_\gamma \subseteq \pi_G^{-1}[U]$ and so $\lim D = x$. Moreover, it is easy to realize that actually D is strictly convergent to x. But then $x \in B^{sr}$and we have a contradiction with $x_F = \pi_F(x) \notin \pi_f[B^{sr}]$. $\Diamond$

Remark. The theorem above is in general the best we can hope for, as even the product of two compact radial spaces may fail to be semi-radial [8].

This suggests to look for possible conditions which may guarantee the semi-radiality of a product. In this respect, we have the following general result:

Theorem 2. *If a class of compact semi-radial spaces is finitely productive then it is countably productive.*

Proof. Let $\{X_n : n \in \omega\}$ be a subfamily of some finitely productive class of compact semi-radial spaces and put $X = \prod_{n \in \omega} X_n$.

Let π_n be the projection of X onto the product of the first $n+1$ factors and let A be a non-κ-closed subset of X. Obviously, we may assume that κ is minimal, i.e. that A is $< \kappa$-closed. Choose a point $x \in \overline{A} \setminus A$. If $\pi_n(x) \in \pi_n[A]$ for every n, then we may pick points $x_n \in A$ in such a way that $\pi_n(x_n) = \pi_n(x)$ for every n. It is clear that the sequence $\{x_n : n \in \omega\}$ converges to x and we are done because $\omega \leq \kappa$.

Now, assume that there is some integer m so that $\pi_m(x) \notin \pi_m[A]$ and consequently $\pi_m[A]$ is not closed in $\pi_m[X]$. Thanks to the finite productivity of our class of spaces, we can assume, without any loss of generality, that $m = 0$.

It is clear that $\pi_0[A]$ is not κ-closed and $< \kappa$-closed and so there exists a sequence $\{x_{0,\alpha} : \alpha \in \kappa\} \subseteq A$ such that the sequence $\{\pi_0(x_{0,\alpha}) : \alpha \in \kappa\}$ converges to a point $x'_0 \in X_0 \setminus \pi_0[A]$. Obviously κ must be regular.

If $\kappa = \omega$ then, taking into account that X is sequentially compact, we can fix a subsequence $\{y_n : n \in \omega\} \subseteq \{x_{0,n} : n \in \omega\}$ converging to a point $y \in X$. Since we must have $\pi_0(y) = x'_0 \notin \pi_0(A)$, we see that $y \notin A$ and we reach our target.

Then, suppose $\kappa > \omega$ and let $D_\alpha = \overline{\{\pi_1(x_{0,\beta} : \beta \in \alpha\}} \subseteq \pi_1[A]$ and $D = \cup_{\alpha\in\kappa} D_\alpha$. Since D is not closed in the semi-radial space $X_0 \times X_1$, there exists a sequence $\{y_\alpha : \alpha \in \lambda\} \subseteq D$ converging to a point $y \in \overline{D} \setminus \setminus D$. It is clear that we should have $\lambda = \kappa$ and $\pi_0(y) = x'_0$. Next, choose points $x_{1,\alpha} \in A$ in such a way that $\pi_1(x_{1,\alpha}) = y_\alpha$ and let $y = (x'_0, x'_1)$.

By repeating this argument, we see that we can define for every $n \in \omega$ a sequence $\{x_{n,\alpha} : \alpha \in \kappa\} \subseteq A$ in such a way that the sequence $\{\pi_n(x_{n,\alpha}) : \alpha \in \kappa\}$ converges to the point $(x'_0, x'_1, \dots, x'_n) \notin \pi_n[A]$.

Let x' be the point of X whose nth coordinate is x'_n. For every α select a point x'_α which is the limit of a convergent subsequence of the set $\{x_{n,\alpha} : n \in \omega\}$. It is clear that the point x'_α belongs to A for every α. The sequence $\{x'_\alpha : \alpha \in \kappa\}$ converges to x'. To check this, it is sufficient to consider neighbourhoods of x' of the form $\pi_m^{-1}[U]$, where U is a closed neighbourhood of $\pi_m(x')$ in $\pi_m[X]$. By construction, for every $n \geq m$ there exists an ordinal $\alpha_n \in \kappa$ such that $\pi_n(x_{n,\alpha}) \in U \times \times X_{m+1} \times \cdots \times X_n$ whenever $\alpha \geq \alpha_n$. Let $\hat{\alpha} = \sup\{\alpha_n : n \geq m\}$. For any $\alpha \geq \hat{\alpha}$ and any $n \geq m$ we have $x_{n,\alpha} \in \pi_m^{-1}[U]$ and therefore $x'_\alpha \in \pi_m^{-1}[U]$. This shows that the sequence $\{x'_\alpha : \alpha \in \kappa\}$ converges to $x' \in \overline{A} \setminus A$. The proof is then complete. ◇

A good class of spaces satisfying the hypothesis of Th. 2 is the class of compact R-monolithic spaces. In fact such class is itself countably productive [2].

The question now is whether some uncountable product can be semi- radial. A result in this direction is

Proposition 2 [1]. *If $\mathfrak{p} > \omega_1$ then the product of ω_1 compact LOTS is semi-radial.*

In view of Th. 2, we may consider the product of ω_1 compact semi-radial spaces. The obstacle here is in the separable subsets. We have:

Theorem 3. *Let $\mathcal{X}$ be a finitely productive class of compact semi-radial spaces (for instance, the class of compact R-monolithic spaces) and let $X = \prod_{\alpha\in\omega_1} X_\alpha$ be a product of members of $\mathcal{X}$. If every sequentially closed separable subset of X is closed then X is semi-radial.*

Proof. Since every sequentially closed separable subset of X is closed, we see that every non-ω-closed subset of X has a countable sequence converging outside it. Thus, we need to work on the non-λ-closed subset with $\lambda > \omega$.

Let then A be a non-λ-closed subset of X, which we can clearly assume to be also $< \lambda$-closed. Denote by $\pi_\beta : \prod_{\alpha\in\omega_1} X_\alpha \to \prod_{\alpha\in\beta} X_\alpha$ the canonical projection and select a point $x \in \overline{A} \setminus A$. If $\pi_\beta(x) \in \pi_\beta[A]$ for each $\beta \in \omega_1$, then pick points $x_\beta \in A$ in such a way that $\pi_\beta(x) = \pi_\beta(x_\beta)$. It is clear that the sequence $\{x_\beta : \beta \in \omega_1\}$ converges to x and therefore we have found in A a sequence of length at most λ which converges to a point outside A.

Next we assume that for some $\beta \in \omega_1$ $\pi_\beta(x) \notin \pi_\beta[A]$. This means that $\pi_\beta[A]$ is a non λ-closed subset of $\prod_{\alpha\in\beta} X_\alpha$ which moreover is $< \lambda$-closed. By Th. 2, the space $\prod_{\alpha\in\beta} X_\alpha$ is semi-radial and so there exists a sequence $\{x_\alpha^\beta : \alpha \in \lambda\} \subseteq A$ and a point $x^\beta \in \prod_{\alpha\in\beta} X_\alpha \setminus \pi_\beta[A]$ such that the sequence $\{\pi_\beta(x_\alpha^\beta) : \alpha \in \lambda\}$ converges to x^β. Clearly λ must be regular. Let $C_\gamma = \overline{\{x_\alpha^\beta : \alpha \in \gamma\}}$ and $C = \cup_{\gamma\in\lambda} C_\gamma$. Observe that $\pi_{\beta+1}[C]$ is a $< \lambda$-closed non λ-closed subset of $\prod_{\alpha\in\beta+1} X_\alpha$. Again by Th. 2, there exists a sequence $\{x_\alpha^{\beta+1} : \alpha \in \omega_1\} \subseteq C$ and a point $x^{\beta+1} \in \prod_{\alpha\in\beta+1} X_\alpha$ such that $\{\pi_{\beta+1}(x_\alpha^{\beta+1}) : \alpha \in \lambda\}$ converges to $x^{\beta+1}$. Furthermore, the sequence $\{x_\alpha^{\beta+1} : \alpha \in \lambda\}$ can be chosen in such a way that $x_\gamma^{\beta+1} \notin C_\gamma$ for every $\gamma \in \lambda$ and consequently $x_\gamma^{\beta+1} \in \overline{\{x_\alpha^\beta : \gamma \in \alpha \in \lambda\}}$.

It is easy to see that the sequence $\{\pi_\beta(x_\alpha^{\beta+1}) : \alpha \in \lambda\}$ must converge to x^β and hence $x^{\beta+1}$ is an extension of x_β, i.e. $x^{\beta+1}(\alpha) = x^\beta(\alpha)$ for every $\alpha \in \beta$. Iterating this procedure, we construct sequences $\{x_\alpha^{\beta+n} : \alpha \in \lambda\} \subseteq A$ in such a way that $\{\pi_{\beta+n}(x_\alpha^{\beta+n}) : \alpha \in \lambda\}$ converges to $x^{\beta+n} \in \prod_{\alpha\in\beta+n} X_\alpha$, $(*)$ $x_\gamma^{\beta+n+1} \in \overline{\{x_\alpha^{\beta+n} : \gamma \in \alpha \in \lambda\}}$ and $x^{\beta+n+1}$ is an extension of $x^{\beta+n}$. At the limit step $\beta+\omega$, define $x^{\beta+\omega} = \cup_{n\in\omega} x^{\beta+n}$ and for any $\alpha \in \lambda$ pick a point $x_\alpha^{\beta+\omega} \in \overline{\{x_\alpha^{\beta+n} : n \in \omega\}} \subseteq A$. It is easy to check that the sequence $\{\pi_{\beta+\omega}(x_\alpha^{\beta+\omega}) : \alpha \in \lambda\}$ converges to $x^{\beta+\omega}$.

Mimicking the same pattern, we can construct for any $\gamma \in \omega_1$ sequences $\{x_\alpha^{\beta+\gamma} : \alpha \in \lambda\} \subset A$ and points $x^{\beta+\gamma} \in \prod_{\alpha\in\beta+\gamma} X_\alpha$ in such a way that $\{\pi_{\beta+\gamma}(x_\alpha^{\beta+\gamma}) : \alpha \in \lambda\}$ converges to $x^{\beta+\gamma}$ and $x^{\beta+\delta}$ is an extension of $x^{\beta+\gamma}$ whenever $\gamma \in \delta$. Furthermore we require that a condition analogous to $(*)$ holds for every successor ordinal and for

γ limit $x_\alpha^{\beta+\gamma}$ is chosen in the set $\bigcap\{\overline{\{x_\alpha^{\beta+\delta} : \xi \in \delta \in \gamma\}} : \xi \in \gamma\}$. To finish put $z = \cup_{\gamma\in\omega_1} x^{\beta+\gamma} \in \prod_{\alpha\in\omega_1} X_\alpha \setminus A$. We distinguish two cases. If $\lambda = \omega_1$ then put $z_\alpha = x_\alpha^{\beta+\alpha}$ for each $\alpha \in \omega_1$. We claim that the sequence $\{z_\alpha : \alpha \in \omega_1\}$ converges to z. To this end fix a neighbourhood W of z. Without any loss of generality, we can take $W = \pi_{\beta+\gamma}^{-1}(V)$ where V is a closed neighbourhood of $x^{\beta+\gamma}$ in $\prod_{\alpha\in\beta+\gamma} X_\alpha$. Clearly there exists $\hat{\alpha} \in \omega_1$ such that $x_\alpha^{\beta+\gamma} \in \pi_{\beta+\gamma}^{-1}(V)$ for any $\alpha \geq \hat{\alpha}$. Since by construction for any $\alpha \geq \hat{\alpha}$, we have $x_\alpha^{\beta+\gamma+1} \in \overline{\{x_\xi^{\beta+\gamma} : \hat{\alpha} \in \xi \in \omega_1\}}$, we see that $x_\alpha^{\beta+\gamma+1} \in \pi_{\beta+\gamma}^{-1}(V)$. With the same argument, we deduce that $x_\alpha^{\beta+\gamma+n} \in \pi_{\beta+\gamma}^{-1}(V)$ for any $\alpha \geq \hat{\alpha}$ and any $n \in \omega$. But then $x_\alpha^{\beta+\gamma+\omega} \in \pi_{\beta+\gamma}^{-1}(V)$ and continuing in this manner we obtain that $x_\alpha^{\beta+\delta} \in \pi_{\beta+\gamma}^{-1}(V)$ for any $\alpha \geq \hat{\alpha}$ and any $\gamma \leq \delta \in \omega_1$. Now replacing $\hat{\alpha}$ with $\max\{\hat{\alpha}, \gamma\}$, we see that $z_\alpha = x_\alpha^{\beta+\alpha} \in W$ for any $\alpha \geq \hat{\alpha}$ and therefore $\{z_\alpha : \alpha \in \omega_1\}$ converges to z.

If $\lambda > \omega_1$, then for any $\alpha \in \lambda$ pick a complete accumulation point z_α of the set $\{x_\alpha^{\beta+\gamma} : \gamma \in \omega_1\}$. It is evident that $z_\alpha \in A$ for every $\alpha \in \lambda$. We claim that $\{z_\alpha : \alpha \in \lambda\}$ converges to z. Fix as before a closed neighbourhood of z of the form $W = \pi_{\beta+\gamma}^{-1}(V)$. For every $\delta \geq \gamma$ there exists $\alpha_\delta \in \lambda$ such that $\pi_{\beta+\delta}(x_\alpha^{\beta+\delta}) \in \pi_{\beta+\delta}(W)$ whenever $\alpha \geq \alpha_\delta$.As λ is regular we can select $\hat{\alpha} \in \lambda$ such that $\alpha_\delta \in \hat{\alpha}$ for every $\delta \in \omega_1$. We have $x_\alpha^{\beta+\delta} \in W$ for every $\delta \in \omega_1$ and every $\hat{\alpha} \in \alpha \in \lambda$ and therefore $z_\alpha \in W$ for every $\hat{\alpha} \in \alpha \in \lambda$. Again we see that the sequence $\{z_\alpha : \alpha \in \lambda\}$ converges to z and this completes the proof. $\Diamond$

It is easy to see that Prop. 2 is an immediate consequence of Th. 3.

The product of ω_1 compact R-monolithic spaces would be consistently semi-radial if the following question had a positive answer.

Question 1. *Is there any model of ZFC where any sequentially closed separable subset of a product of ω_1 compact sequential spaces is always closed?*

We wonder if [PFA] could be of any help in solving such question.

Another thing emerges by comparing Th. 1 with the result of E. Murtinová. In fact, we may ask:

Question 2. *Is it true that the product of ω_1 compact semi-radial spaces is almost radial if it is sequentially compact?*

We do not know the answer even for the very simple case of the space I^{ω_1}.

It is well known that I^{ω_1} is pseudoradial if and only if $\mathfrak{s} > \omega_1$ and that there are models where I^{ω_1} is pseudoradial but not semi-radial (take, for instance, a model of $2^{\omega_1} > 2^{\omega} = \omega_2 = \mathfrak{s}$, [4]). Thus, we have:

Question 2′. *Is it true that I^{ω_1} is almost radial if $\mathfrak{s} > \omega_1$?*

References

[1] A. BELLA, A.: Few remarks and questions on pseudoradial spaces, *Topology Appl.* **70** (1996), 113–123.

[2] BELLA, A. and DOW, A.: On R-monolithic spaces, *Topology Appl.*, to appear.

[3] BELLA, A., DOW, A. and TIRONI, G.: Pseudoradial spaces: separable subsets, products and maps onto Tychonoff cubes, *Topology Appl.*, to appear.

[4] van DOUWEN, E. K.: The integers and topology, Handbook of Set-Theoretic Topology, K. Kunen and J. E. Vaughan editors, North-Holland, Amsterdam, The Netherlands, 1984, 111–167.

[5] MALYKHIN, D. V.: A product of countably many compact semi-radial spaces is pseudoradial, to appear.

[6] MURTINOVÁ, E.: On products of pseudoradial spaces, to appear.

[7] OBERSNEL, F. and TIRONI, G.: Product of pseudoradial spaces, *Mathematica Pannonica* **6** (1995), 29–38.

[8] SIMON, P. and TIRONI, G.: Pseudoradial spaces: finite products and an example from CH, *Serdica Math. J.* **24** (1998), 127–134.