Mathematica Pannonica
8/2 (1997), 165–172

IN MEMORIAN MARIO DOLCHER

Gino **Tironi**

Department of Mathematical Sciences, University of Trieste, Piazzale Europa 1, I–34100, Trieste, Italy

Professor Mario Dolcher, member of the Editorial Board of the journal Mathematica Pannonica, died because of an ictus on July 10, 1997 in Sassari (Sardinia), where he was visiting his brother. He was born on January 22, 1920 in Zara (in Italy at the time, now Zadar in Croatia). He attended the gymnasium in Gorizia and entered the Scuola Normale Superiore in Pisa where he graduated on June 30, 1942 under the direction of Professor Leonida Tonelli. In the Academic Year 1942–1943 he perfectioned his studies at the Eidgenössische Technische Hochschule in Zürich under the direction of Professor Heinz Hopf. He began the academic activity in the University of Trieste on April 1, 1945, when the war was not yet over, devoting his teaching and research work first to the Faculty of Engineering, since the Faculty of Sciences was activated some years later. He was one of the first collaborators of Professor Bruno de Finetti, the first chairman, in 1947, of the Institute of Mathematics (formerly Seminar of Mathematics, founded in 1946 by Professor Ugo Morin and directed from 1947 by de Finetti). He taught in Trieste courses on "Higher Analysis", "Complements of Mathematics", "Mathematical Analysis" from the Academic Year 1948–1949 to 1950–1951. He then was forced to move to the University of Ferrara, where on January 1956 he obtained the "Libera Docenza" in Topology, the first in Italy. He came back to Trieste in the Academic Year 1957-1958 and taught courses such as "Topology", "Elementary Mathematics from a higher Standpoint", "Higher Mathematics", "Algebra". On February 1, 1965 he was appointed "Professore Ordinario" (Full Professor) of "Mathematical Analysis" at the Faculty of Engineering of the University of Trieste. He kept the chair till he retired in November 1995. After the unanimous proposal of the Faculty on July 10, 1995, the Italian Minister of University and Scientific and Technological Research nominated him "Professor Emeritus" on December 7, 1995.

The **scientific work** of Professor Mario Dolcher had a great influence for many years on teaching of many mathematicians in Trieste. He suggested lots of interesting problems in Topology, but also in different fields of Mathematics. Several members of the Department of Mathematical Sciences were mathematically inspired by him, as invaluable teacher in courses of Analysis, Algebra, Topology and Elementary Mathematics from an Higher Standpoint or as supervisor of their dissertations. Some of them, after a "topological" beginning, changed their mathematical interests and are now working in fields ranging from Numerical Analysis to Measure Theory, to Differential Equations, to Mathematical Economics, to Theoretical Informatics, but all preserved the love for the rigour and honesty of the proof and for the mathematical fantasy and invention that was a typical character of the work of Mario Dolcher. He has always been for us a model to follow, sometimes difficult to imitate. Almost all Dolcher's papers were written in Italian; in addition some results, even important ones, had a very limited circulation (in fact they were published as "Quaderni matematici", i.e. internal reports of the Trieste Mathematical Institute), so the most important ones were discovered late by the international scientific community.

Initially Dolcher studied the properties of continuous transformations of a plane (a complex plane) into itself. The main idea was to single out those properties of analytic functions depending only on the topology of the Riemann surface and hence depending only on the continuity of the transformations of a sphere into itself and not on the metric properties (monogeneity) of the function, following the methods of T. Radó [12] and L. Cesari [3]. To this study are dedicated papers (1), (2), (5), (6), (7), (9), (10). In (3) he presented the notion of a mathematical structure, in which some criticism was raised to the Bourbaki's approach. This was firmly opposed by J. Dieudonné [4], while quite different was the opinion of A. Bennet [2]. (18) is the last answer and view on the subject. In (4) Dolcher solved problems concerning the existence:

(a) of minimal closed subspaces separating a given pair of points in a topological space;

(b) of minimal closed subspaces that disconnect the space;
(c) of a minimal closed subset contained in any closed set separating a given pair of points.

In the case of locally connected spaces, necessary and sufficient conditions were given, which were shown to be sufficient in the case of a general topological space. These results extended to a (locally connected) space similar results proved by Mazurkiewicz [9], [10] and Kuratowski [8] in the case of the space $\mathbb{R}^n$.

In 1955 Dolcher began to be interested in aspects of General Topology, which eventually became his main research interest and the starting point to research of many of his students. In (8), in order to obtain a general method of extension of topological spaces (in particular, compactifications and completions), he introduced the notion of a topological space associated with any family of filters. This method was first applied to the family of compactifying filters by his student Gerolini [6].

In (11) it is shown that *given an r-tree A, if C and C' are two countable families of chains that cover A then there is an automorphism of A which transforms the family C in the family C'*. As a consequence, it is proved that

Theorem. *There is a countable, totally disconnected Hausdorff space without isolated points, which is not isomorphic to the rational numbers $\mathbb{Q}$.*

The most widely known paper of Dolcher and the one that have had more success among his followers is (12): Topologie e strutture di convergenza. There, a thorough study of the interrelations between convergence structures on a set and topologies on the same set is fulfilled. In particular, two functors $L : \mathcal{T} \to \mathcal{L}$ (from topological spaces to convergence structures) and $T : \mathcal{L} \to \mathcal{T}$ (in the opposite direction) are defined. L associates to every topological space (E, τ) a convergence on E as follows: A sequence in E $L(\tau)$-converges to $p \in E$ if it is eventually in every neighborhood of p. Given a convergence structure λ on a set E a topology $T(\lambda)$ on E is defined as follows: A set $A \subset E$ is open in $T(\lambda)$ if and only if for every $p \in A$ every sequence converging to p is eventually in A (i.e. A is sequentially open). The functors L and T are defined in the obvious way on the maps. The two functors are shown to be monotone. It is shown that a topology τ is deducible from a convergence (i.e. $\tau = T(\lambda)$) if and only if

$$TL(\tau) = \tau.$$

Analogously a convergence λ is deducible from a topology if and only if

$$LT(\lambda) = \lambda.$$

There are convergences which are not deducible from a topology, but all convergences having unicity of the limit (the class of convergences called $\mathcal{L}_2$) are; if they are deducible from a topology, in general, they are deducible from infinitly many topologies and $T(\lambda)$ is the finer one. For topologies, it is proved that not all topologies, even not all Hausdorff topologies, can be deduced from a convergence. However if τ is a Hausdorff deducible topology it can be deduced from a unique convergence. The behaviour of the sequential closure is also examined. In Internal Reports (IR) (3), answering a problem raised by J. Novák [11], Dolcher shows that for any $\eta \leq \omega_1$ there is a space having sequential degree η. A model E_η of such a space is produced for each η and the cardinality of such model is $\aleph_0$ for $0 < \eta < \omega$; it is $2^{\aleph_0}$ for $\omega \leq \eta < \omega_1$; it is $2^{\aleph_1}$ for $\eta = \omega_1$. The problem of characterizing the topologies deducible from a convergence (i.e. the sequential topologies) is examined once more in IR (4) where the following theorem is proved

Theorem. *A topology τ on a set E satisfies the condition $TL(\tau) =$ $= \tau$ (i.e. it is a sequential topology) if and only if for every $x \in E$ the* enlarged envelope $[x]^{\Phi,\omega^*}$ *coincides with the neighborhood filter σ_x of x.*

In (17) the pseudoradial spaces are characterized in an interesting way. It is known that pseudoradial topological spaces are a generalization of sequential spaces (see for example [7], [1], [5]). Here a sequence means any map from an ordinal number η to a set E. A topological space is said to be pseudoradial if for any non-closed subset $C \subset E$ there is a point $x \in \overline{C} \setminus C$ and a sequence $(x_\alpha)_{\alpha<\eta}$ such that $x_\alpha \in C$ for any $\alpha < \eta$ and $x_\alpha \to x$. Dolcher introduced the following definition of a colimit of a filter $\mathcal{G}$ according to a family of filters Φ. Given a set E, a filter $\mathcal{G}$ and a family Φ of filters $\mathcal{F}_x$ indexed by the elements of E, for $M \subset E$ consider the filter $\mathcal{F}_M = \bigwedge_{x\in M} \mathcal{F}_x$, generated by sets $\cup_{x\in M} F_x, F_x \in \mathcal{F}_x$. The family of sets

$$\left\{ \bigcup_{n=0}^{\infty} A_n : A_0 \in \mathcal{G}, A_{n+1} \in \mathcal{F}_{A_n}, n \in \mathbb{N} \right\}$$

is a filter base. The filter generated by it is said to be the colimit of $\mathcal{G}$ according to Φ and is denoted by

$\mathrm{Col}_{\Phi}\mathcal{G}$.

The case of interest here is the following: E is a topological space, $\mathcal{G}$ is the filter $[A]$, with $\emptyset \neq A \subset E$, possibly $A = \{x\}$, $\mathcal{F}_x$ is the intersection of all sequence filters converging to x. The following theorem is proved

Theorem. *A topological space X is pseudoradial if and only if for every point x the filter of its neighborhoods is the colimit of the ultrafilter $[x]$.*

His books and notes on Mathematical Analysis, Algebra and Topology strongly influenced generations of students of Mathematics, Physics and Engeneering. His lectures on Elementary Mathematics from a Higher Standpoint benefited many future teachers.

The following is a list, complete to the best of my knoledge, of the works of Professor Mario Dolcher.

Publications of Mario Dolcher

Papers published in journals

(1) Geometria delle trasformazioni continue, I: Sopra un teorema di Radó, *Annali della Scuola Normale Superiore di Pisa* (Serie II) **14** (1945), 99–116.

(2) Due teoremi sull'esistenza di punti uniti nelle trasformazioni piane continue, *Rendiconti Sem. Matem. Univ. Padova* **17** (1948), 97–101.

(3) Nozione generale di struttura per un insieme, *Rend. Sem. Matem. Univ. Padova* **18** (1949), 265–291.

(4) Questioni di minimo per insiemi chiusi sconnettenti uno spazio topologico, *Rend. Sem. Matem. Univ. Padova* **19** (1950), 159–171.

(5) Geometria delle trasformazioni continue. Un rafforzamento di enunciati precedenti, *Rivista Matem. Università di Parma* **2** (1951), 331–335.

(6) Geometria delle trasformazioni continue. Un teorema sulle trasformazioni di varietà semplici n-dimensionali, *Ann. Univ. Ferrara* (Sez. VII N.S.) **3** (1954), 11–16.

(7) Geometria delle trasformazioni continue. Un teorema sulle trasformazioni di una corona circolare, *Rivista Matem. Università di Parma* **5** (1954), 339–361.

(8) Topologia delle famiglie di filtri, *Rendiconti Sem. Matem. Univ. Padova* **24** (1955), 443–473.

(9) Geometria delle trasformazioni continue. Sul numero delle immagini inverse nelle trasformazioni di un dominio piano pluriconnesso, *Ann. Univ. Ferrara* (Sez. VII N.S.) **4** (1954–1955), 1–7.

(10) Determinazione del massimo numero di eccezioni alla n-vocità dell'inversa di una trasformazione piana continua, *Annali Univ. Ferrara* (Nuova Serie) **7** (1957–1958), 53–64.

(11) Una proprietà delle famiglie numerabili di catene sopra un albero, *Ann. Univ. Ferrara* (Sez. VII N. S.) **8** (1958–1959), 7–16.

(12) Topologie e strutture di convergenza, *Annali Scuola Normale Sup. di Pisa* (Serie III) **14** (1960), 63–92.

(13) Su un criterio di convergenza per le successioni monotone di funzioni quasi-periodiche, *Rend. Sem. Mat. Univ. Padova* **34** (1964), 191–199.

(14) Questioni topologiche collegate con la quasi-periodicità nel campo reale, *Rend. Sem. Matem. Univ. Padova* **35** (1965), 1–17.

(15) Sulla convergenza puntuale delle funzioni, *Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur.* (VIII) **43** (1967), 300–304.

(16) Deducibility of a topology of the convergence of sequences, *Rend. Sem. Mat. Univ. Politec. Torino* **41** (1983), 31–37.

(17) An alternative definition of pseudoradial spaces, Convergence Structures 1984, Proc. of the Conference on Convergence, Ed. by J. Novák, W. Gähler, H. Herrlich, J. Mikusiński, Akademie-Verlag, Math. Res. **24** Berlin 1985, 71–76.

(18) Some ideas on structures, Proc. Eleventh International Conference of Topology (Trieste, 1993), Rend. Istit. Mat. Univ. Trieste **25** (1993), 489–495.

Internal reports

(1) Topologia, Centro Internazionale Matematico Estivo (C.I.M.E.) 3^o Ciclo–Varenna, Villa Monastero: 26 agosto – 3 settembre 1955. (Lectures of K. Kuratowski, G. Scorza-Dragoni, E. Sperner, G. Darbo, M. Dolcher, M. Vaccaro), Roma, Istituto Matematico dell'Università, (1955–1956).

(2) Sul problema dell'approssimazione per le funzioni quasi-periodiche in un gruppo abeliano topologico, *Quaderni matematici, Istituto di Matematica, Univ. di Trieste*, 1963–64.

(3) Esistenza di strutture di convergenza aventi grado successionale arbitrario, *Quaderni matematici, Istituto di Matematica, Univ. di Trieste* (No date, but 1966–1967 is likely).

(4) Condizioni per la deducibilità di una topologia da convergenza di successioni, *Quaderni matematici, Istituto di Matematica, Univ. di Trieste* (No date, but 1966–1967 is likely).

Books and typewritten notes

(1) Analisi matematica. Fascicolo I. Nozioni sugli insiemi, Tipografia Moderna-Trieste-Editrice (1967–1968).

(2) Analisi matematica. Fascicolo IV. Funzioni, Tipografia Moderna-Trieste-Editrice (1967–1968).

(3) Analisi matematica. Fascicolo V. Calcolo differenziale, Tipografia Moderna-Trieste-Editrice (1967–1968).

(4) Analisi matematica. Fascicolo VI. Polinomi; Numeri complessi, Tipografia Moderna-Trieste-Editrice (1967–1968).

(5) Analisi matematica. Fascicolo II. Calcolo combinatorio; Probabilità, Tipografia Moderna-Trieste-Editrice (1970–1971).

(6) Analisi matematica. Fascicolo III. Numeri reali, Tipografia Moderna-Trieste-Editrice (1972–1973).

(7) Analisi matematica. Fascicolo VII. Serie a termini reali, Tipografia Moderna-Trieste-Editrice (1973–1974).

(8) Global Analysis and its applications, Voll. I–II–III. Lectures presented at the International Seminar Course held at the International Center fot Theoretical Physics, Trieste, from 4 July to 25 August 1972; Edited by P. de la Harpe, M. Dolcher, J. Eells and E.C. Zeeman, International Atomic Energy Agency, Vienna, 1974.

(9) Algebra lineare, Zanichelli, Bologna (1978), pp. vi + 122.

(10) Elementi di Analisi matematica, Voll. I - II. Edizioni LINT, Trieste, First Edition 1991. pp. xv + 349 (Vol. I), 514 (Vol II).

References

[1] ARHANGEL'SKIĬ, A.V.: Structure and classification of topological spaces and cardinal invariants, *Russian Math. Surveys* **33** (1978), 33–95.

[2] BENNET, A.: (review) *Journ. of Symbolic Logic* (1950).

[3] CESARI, L.: Su un teorema di Radó sulle trasformazioni continue, *Atti Ist. Veneto Scienze e Lettere* **C** (1942).

[4] DIEUDONNÉ, J.: (review) *Math. Reviews* (1950), 238.

[5] DIMOV, G. ISLER, R. and TIRONI, G.: On functions preserving almost radiality and their relations to radial and pseudoradial spaces, *Comm. Math. Univ. Carol.* **28** (1987), 751–781.

[6] GEROLINI, A.: Compactification des espaces séparés, *C. R. Acad. Sci. Paris* **232** (1951), 1056–1058.

[7] HERRLICH, H.: Quotienten geordneter Räume und Folgenkonvergenz, *Fund. Math.* **61** (1967),

[8] KURATOWSKI, C.: Sur les coupures irréductible du plan, *Fund. Math.* **6** (1924).

[9] MAZURKIEWICZ, S.: Sur un ensemble G_δ ponctiforme qui n'est homéomorphe à aucun ensemble linéaire, *Fund. Math.* **1** (1920)

[10] MAZURKIEWICZ, S.: Sur les continus plans non bornés, *Fund. Math.* **5** (1924).

[11] NOVÁK, J.: On some problems concerning multivalued convergence, *Czech. Math. Journ.* **14** (1965) 74–100.

[12] RADÓ, T.: On continuous transformations in the plane, *Fund. Math.* **27** (1930).