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CORRIGENDUM TO "PRODUCTS OF PSEUDORADIAL SPACES"

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In our paper (Math. Pannonica **6**/1 (1995), 29 – 38) the proofs of Ths. 3.2 and 3.3 are wrong. They should be substituted as follows; in order to make as little changes as possible Ths. 3.2 and 3.3 will be labeled 3.3 and 3.3′, while the next Th. 3.2 is a proposition of independent interest.

Theorem 3.2. *Let* $(X_\alpha)_{\alpha\in\omega_1}$ *be a family of compact Hausdorff spaces and for each* $\alpha \in \omega_1$ *let* $|X_\alpha| < 2^{\omega_2}$. *Then every closed subset* F *of the cartesian product* $\prod_{\alpha\in\omega_1} X_\alpha$ *has a point* p *such that* $\chi(p, F) \le \omega_1$.

Proof. Let F be a closed subset of $X = \prod_{\alpha\in\omega_1} X_\alpha$. Let $\pi_\alpha: X \to X_\alpha$ be the projection on the α-th factor of the product. Since $|X_0| < 2^{\omega_2}$, then, by the Čech – Pospišil theorem, there is a point $x_0 \in \pi_0(F) \subset X_0$ such that $\chi(x_0, \pi_0(F)) \le \omega_1$. Let $F_1 = (\{x_0\} \times \prod_{1\le\alpha<\omega_1} X_\alpha) \cap F$. Clearly F_1 is a G_{ω_1} set in $F = F_0$ and there is $x_1 \in \pi_1(F_1) \subset X_1$ such that $\chi(x_1, \pi_1(F_1)) \le \omega_1$.

Suppose we have found points and closed sets x_γ and F_γ, where $x_\gamma \in \pi_\gamma(F_\gamma) \subset X_\gamma$ such that F_γ is a G_{ω_1} subset of F for every $\gamma < \alpha$, $\alpha < \omega_1$ and $F_\gamma \subset F_{\gamma'}$ whenever $\gamma' \le \gamma$. If α is limit, take $F_\alpha = \cap_{\gamma<\alpha} F_\gamma$. By compactness $F_\alpha \ne \emptyset$.

If $\alpha = \delta + 1$ define $F_\alpha = \big((x_\gamma)_{\gamma \leq \delta} \times \prod_{\alpha \leq \beta < \omega_1} X_\beta\big) \cap F$. In both cases F_α is a G_{ω_1} subset of F and, moreover, we can select a point $x_\alpha \in \pi_\alpha(F_\alpha) \subset X_\alpha$ such that $\chi(x_\alpha, \pi_\alpha(F_\alpha)) \leq \omega_1$. Now the point $p = (x_\alpha)_{\alpha < \omega_1}$ satisfies $\{p\} = \bigcap_{\alpha < \omega_1} F_\alpha$ and therefore it is a G_{ω_1} point in F. This shows that $\chi(p, F) \leq \omega_1$. ◊

This theorem can be considered as a generalization of the Čech – Pospišil theorem in the case that the compact space X is given as a product of a family of compact spaces.

Theorem 3.3. *Let $\mathcal{F} = (X_n)_{n < \omega_0}$ be a family of Hausdorff compact pseudoradial spaces and $|X_n| < 2^{\omega_2}$ for every $n < \omega_0$. Then $\prod \mathcal{F}$ is pseudoradial.*

Proof. In fact, by the previous Th. 3.2, if F is a closed subset of the product space, then there is a point $p \in F$ such that $\chi(p, F) \leq \omega_1$. Each of the X_n is a CSC space, and also $\prod \mathcal{F}$ is a CSC space. So, by Th. 3.1, it is also a pseudoradial space. ◊

Concerning the equality $\mathfrak{h} = \mu$, it should be remarked that it was completely proven only recently in [1].

The following (consistently) more general result can then be proved

Theorem 3.3′. *Suppose that $\mathfrak{h} \leq \omega_2$ holds. Then, if $\mathcal{F}$ is a family of strictly less than $\mathfrak{h}$ compact Hausdorff pseudoradial spaces each one having cardinality $< 2^{\omega_2}$, the cartesian product $\prod \mathcal{F}$ is pseudoradial.*

Proof. In fact, the proof follows the same scheme as in the previous theorem. If F is a closed subset of $\prod \mathcal{F}$ there is a point $p \in F$ such that $\chi(p, F) \leq \omega_1$, since $|\mathcal{F}| < \mathfrak{h} \leq \omega_2$ means $|\mathcal{F}| \leq \omega_1$, and the product is still a CSC space; so, by Th. 3.1, it is pseudoradial. ◊

Obviously if $\mathfrak{c} = \omega_1$ Th. 3.3 reduces to Th. 3.2′.

Reference

[1] SIMON, P.: Products of sequentially compact spaces, Proc. XI Int. Conference of Topology, Trieste (1993), *Rend. Ist. Matem. Univ. Trieste* **25** (1993), 447–450.