Mathematica Pannonica
4/2 (1993), 205 – 215

ON MEASURING THE FUZZINESS AND THE NONFUZZINESS OF INTUITIONISTIC FUZZY SETS

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Received September 1992

AMS Subject Classification: 94 D 05

Keywords: Intuitionistic fuzzy sets, fuzziness measure.

Abstract: The present paper aims at proposing two different approaches to the measurement of the nonfuzziness and the fuzziness of an intuitionistic fuzzy set. The notion of an intuitionistic fuzzy set was introduced by K. Atanassov and S. Stoeva [3] as a generalization of the notion of an ordinary Zadeh fuzzy set [13]. The basis of these notions are different approaches to the construction of information theory, namely, the notion of Shannon's entropy [8] and Onicescu's notion of an informational energy [12]. Because of a number of properties which these notions adopted to intuitionistic fuzzy sets possess, they may serve to introduce fuzziness and sharpness measures of those sets.

1. Introduction

Almost contemporaneously with the moment the theory of fuzzy sets [13] and its practical applications appeared, people began thinking about the problem of finding a measure for the "fuzziness" of a fuzzy set. Especially in the construction of various systems, it was important to answer the question which one of two given fuzzy sets is fuzzier and what differences occur between a fuzzy set and the respective ordinary one. This problem was taken up in the seventies by two Italian mathematicians, A. de Luca and S. Termini ([10]), who carried it over to the ground of decision theory.

Let us consider in some space X a fuzzy set characterized by a membership function $f\colon X \to [0,1]$. We assume that X is finite and $X = \{x_1, x_2, \dots, x_n\}$. In the paper [10] A. de Luca and S. Termini introduced some functional $d(f)$, called an entropy of the fuzzy set f. They required that the following conditions hold:

L1. $d(f) = 0 \Leftrightarrow f(x_i) \in \{0,1\}$, $i = 1,2,\dots,n$,
L2. $d(f)$ takes the greatest value $\Leftrightarrow f \equiv \frac{1}{2}$,
L3. $d(f^*) \le d(f)$ where f^* is a "sharper" version of the set f, i.e. when $f^*(x) \ge f(x) \ge \frac{1}{2}$ and $f^*(x) \le f(x) \le \frac{1}{2}$.

Because of such properties, the quantity $d(f)$ may serve for a fuzziness measure of the fuzzy set f. The following definition of the proposed *entropy* $d(f)$ was adopted:

$$d(f) = H(f) + H(f'), \tag{1.1}$$

where $f'(x) = 1 - f(x)$ is the complement of the fuzzy set f, and the function H is of the form $H(f) = -K \sum\limits_{i=1}^{n} f(x_i)\cdot\ln f(x_i)$ (K — constant, $K > 0$). It can be shown that $d(f)$ is a valuation, i.e. that

$$d(f \wedge g) + d(f \vee g) = d(f) + d(g) \tag{1.2}$$

holds for any fuzzy sets f and g (the proof is carried out on the basis of the fact that H is a valuation ([10]; the symbol $\wedge$ denote the minimum and $\vee$ the maximum)). The quantity $d(f)$ may also be written down with the use of Shannon's function as

$$d(f) = K \cdot \sum_{i=1}^{n} S(f(x_i)), \tag{1.3}$$

where $S(x) = -x \ln x - (1-x)\ln(1-x)$. It can also be proved that

$$d(f) = d(f'). \tag{1.4}$$

It was shown in [10] that the entropy $d(f)$ measures the total incertitude connected with the taking of a decision whether to qualify (or not) each element x_i ($i = 1,2,\dots,n$) as a member of the fuzzy set f. It is an equivalent of the probability Shannon entropy and, as such, may serve for a measure of information on some experiment.

In the paper [4] D. Dumitrescu proposed an alternative conception of measuring a fuzzy set. He defined a quantity $E(f)$, called an energy of the fuzzy set f, demanding that it should satisfy the conditions:

D1. $E(f)$ attains its minimum $\Leftrightarrow f \equiv \frac{1}{2}$.
D2. $E(f)$ attains its maximum $\Leftrightarrow f(x_i) = \{0,1\}$, $i = 1, 2, \ldots, n$,
D3. $E(f^*) \geq E(f)$ where f^* is a "sharper" version of the set f (defined in L3).

Because of the above properties, the *energy* $E(f)$ may be treated as a sharpness measure of the fuzzy set f. Dumitrescu adopted the following definition ([4]):

$$E(f) = p(f) + p(f'), \tag{1.5}$$

where $f'(x) = 1 - f(x)$ and $p(f) = \sum_{i=1}^{n} f^2(x_i)$. It can be proved (on the basis of the fact that p is a valuation [4]) that $E(f)$ is a valuation, i.e.

$$E(f \vee g) + E(f \wedge g) = E(f) + E(g), \tag{1.6}$$

and that

$$E(f) = E(f') \tag{1.7}$$

holds. The energy $E(f)$ measures (in contradistinction to the entropy $d(f)$) the quantity of the certitude connected with the taking of a decision whether to assign to elements from the set $X = \{x_1, x_2, \ldots, x_n\}$ an attribute defined by the set f. It is an equivalent of the informational energy proposed by O. Onicescu in [12] as an alternative way of constructing information theory.

Because of the importance of the problems discussed above, we want to propose the analogous quantities for intuitionistic fuzzy sets. The notion of an intuitionistic fuzzy set [1] was for the first time presented by K. Atanassov in 1983 as a generalization of an ordinary fuzzy set ([13]).

By an *intuitionistic fuzzy set* A in some reference set E we mean the structure $A = \{(x, \mu_A(x), \nu_A(x)) : x \in E\}$ where the functions $\mu_A \colon E \to [0,1]$ and $\nu_A \colon E \to [0,1]$, such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$, define, respectively, degrees of the belonging and the non-belonging of the given element x from the set E to the intuitionistic set A. For clarity, we stress that if $\mu_A(x) = 1$, then $\nu_A(x) = 0$ and, conversely, if $\nu_A(x) = 0$, then $\mu_A(x) = 1$. A fuzzy set written down in the convention of an intuitionistic set is the structure $\{(x, \mu(x), 1 - \mu(x))\} \colon x \in E\}$ where $\mu \colon E \to [0,1]$ is its membership function.

According to [3], we define, for intuitionistic sets A and B,

(1.8) $$A \subset B \Leftrightarrow \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \quad \forall x \in E,$$

(1.9) $$A' = \{(x, \mu_{A'}(x), \nu_{A'}(x)) : x \in E\},$$

where $\mu_{A'}(x) = \nu_A(x)$ and $\nu_{A'}(x) = \mu_A(x)$ $(x \in E)$,

(1.10) $$A \cap B = \{(x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) : x \in E\},$$

where $\mu_{A\cap B}(x) = \mu_A(x) \wedge \mu_B(x)$ and $\nu_{A\cap B}(x) = \nu_A(x) \vee \nu_B(x)$ $(\forall x \in E)$,

(1.11) $$A \cup B = \{(x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) : x \in E\},$$

where $\mu_{A\cup B}(x) = \mu_A(x) \vee \mu_B(x)$ and $\nu_{A\cup B}(x) = \nu_A(x) \wedge \nu_B(x)$ $(\forall x \in E)$, the symbol $\wedge$ denoting the minimum, and $\vee$ the maximum. More properties and an example of an intuitionistic set which is not an ordinary fuzzy set can be found in [2]. On account of the possibility of defining some special operations on intuitionistic fuzzy sets, one can show that the class of all such sets satisfies the axioms of von Wright's modal logic ([6]). This is a consequence of intuitionism, a philosophical current which was adopted by L. E. J. Brouwer to mathematics as mathematical intuitionism (see e.g. [9]).

From now on, we assume that the set E is finite and $E = \{x_1, x_2, \ldots, x_n\}$.

2. Arithmetical approach

Let us define in the set E some intuitionistic set $A = \{(x, \mu_A(x), \nu_A(x)) : x \in E\}$.

Definition 2.1. By an *arithmetical entropy* of the intuitionistic set A we mean the quantity

(2.1) $$\tilde{d}(A) = \frac{d(\mu_A) + d(\nu_A)}{2}$$

where $d(\mu_A)$ and $d(\nu_A)$ are the entropies defined by formula (1.1) for fuzzy sets characterized by the functions μ_A and ν_A, respectively.

According to (1.1), the above formula can be written down as

(2.2) $$\tilde{d}(A) = \frac{H(\mu_A) + H(1-\mu_A) + H(\nu_A) + H(1-\nu_A)}{2},$$

whence it is evident that

$$\tilde{d}(A) = 0 \Leftrightarrow \mu_A \equiv 1 \quad (\nu_A \equiv 0) \quad \text{or} \quad \mu_A \equiv 0 \quad (\text{then } \nu_A \equiv 1).$$

Similarly, $\tilde{d}(A)$ takes the greatest value when $d(\mu_A)$ and $d(\nu_A)$ take simultaneously the greatest values, i.e. when $\mu_A \equiv \frac{1}{2}$ and $\nu_A \equiv \frac{1}{2}$ (this means that the intuitionistic set A is "fuzziest"). So, postulates L1 and L2 are satisfied. Similarly, if by A^* we shall mean a "sharper" version of the intuitionistic set A, i.e. an intuitionistic set for which either $\mu_{A^*}(x) \geq \mu_A(x) \geq \frac{1}{2}$ and $\nu_{A^*}(x) \leq \nu_A(x) \leq \frac{1}{2}$ or $\mu_{A^*}(x) \leq \leq \mu_A(x) \leq \frac{1}{2}$ and $\nu_{A^*}(x) \geq \nu_A(x) \geq \frac{1}{2}$, then postulate L3 will also be satisfied. It is easy to show that (1.4) holds. Similarly, (1.2) also holds since the following theorem is true.

Theorem 2.1. *The arithmetical entropy of the intuitionistic set A is a valuation in the class of intuitionistic sets over the space E, that is,*

$$\tilde{d}(A \cup B) + \tilde{d}(A \cap B) = \tilde{d}(A) + \tilde{d}(B). \tag{2.3}$$

Proof. $\tilde{d}(A \cup B) + \tilde{d}(A \cap B) = \frac{d(\mu_{A\cup B})+d(\nu_{A\cup B})}{2} + \frac{d(\mu_{A\cap B})+d(\mu_{A\cap B})}{2} = = \frac{d(\mu_A \vee \mu_B)+d(\nu_A \wedge \nu_B)}{2} + \frac{d(\mu_A \wedge \mu_B)+d(\nu_A \vee \nu_B)}{2} = \frac{d(\mu_A \vee \mu_B)+d(\mu_A \wedge \mu_B)}{2} + + \frac{d(\nu_A \vee \nu_B)+d(\nu_A \wedge \nu_B)}{2} = \frac{d(\mu_A)+d(\mu_B)}{2} + \frac{d(\nu_A)+d(\nu_B)}{2} = \frac{d(\mu_A)+d(\nu_A)}{2} + + \frac{d(\mu_B)+d(\mu_B)}{2} = \tilde{d}(A) + \tilde{d}(B)$ (since d is a valuation in the class of fuzzy sets). ◊

The successive step of our paper is to define the energy of an intuitionistic set.

Definition 2.2. By an *arithmetical energy* $e(A)$ of the intuitionistic set A we mean the quantity

$$e(A) = \frac{E(\mu_A) + E(\nu_A)}{2}, \tag{2.4}$$

where $E(\mu_A)$ and $E(\nu_A)$ are energies (1.5) for fuzzy sets characterized by the functions μ_A and ν_A.

In conformity with (1.5), we may write down that

$$e(A) = \frac{p(\mu_A) + p(1 - \mu_A) + p(\nu_A) + p(1 - \nu_A)}{2}. \tag{2.5}$$

It can easily be noticed that D1 holds since $e(A)$ attains the smallest value when $E(\mu_A)$ and $E(\nu_A)$ are smallest, and this holds if $\mu_A \equiv \frac{1}{2}$ and $\nu_A \equiv \frac{1}{2}$ (then the set A^* is "fuzziest"). It can also be seen that D2 holds, as well as D3, if A is such as the one described earlier. On account of the symmetry of addition, (1.7) also holds. Also (1.6) holds since we have

Theorem 2.2. *The arithmetical energy $e(A)$ of the intuitionistic set A is a valuation in the class of intuitionistic sets over the set E, that is*

$$e(A \cup B) + e(A \cap B) = e(A) + e(B). \tag{2.6}$$

The proof of this theorem is carried out similarly as the proof of

Th. 2.1 by making use of (2.4), (1.10), (1.11) and that E is valuation (see (1.6)).

A consequence of the arithmetical energy of an intuitionistic set, defined in such a way is the possibility of defining a correlation [7] of intuitionistic sets. In accordance with [7], we define an *arithmetical correlation* $c(A, B)$ of the intuitionistic sets A and B as

$$c(A,B) = \frac{\mathsf{C}(\mu_A, \mu_B) + \mathsf{C}(\nu_A, \nu_B)}{2}, \tag{2.7}$$

where

$$\mathsf{C}(\mu_A, \mu_B) = \sum_{i=1}^{n} [\mu_A(x_i) \cdot \mu_B(x_i) + (1 - \mu_A(x_i)) \cdot (1 - \mu_B(x_i))]$$

and

$$\mathsf{C}(\nu_A, \nu_B) = \sum_{i=1}^{n} [\nu_A(x_i) \cdot \nu_B(x_i) + (1 - \nu_A(x_i)) \cdot (1 - \nu_B(x_i))]$$

are correlations of the fuzzy sets μ_A, μ_B and ν_A, ν_B, respectively (see [5]).

In particular, we have

$$c(A, A) = e(A), \tag{2.8}$$

since

$$c(A, A) = \frac{\mathsf{C}(\mu_A, \mu_A) + \mathsf{C}(\nu_A, \nu_A)}{2} =$$

$$= \frac{\sum_{i=1}^{n} [\mu_A^2(x_i) + (1 - \mu_A(x_i))^2] + \sum_{i=1}^{n} [\nu_A^2(x_i) + (1 - \nu_A(x_i))^2]}{2} =$$

$$= \frac{p(\mu_A) + p(1 - \mu_A) + p(\nu_A) + p(1 - \nu_A)}{2} =$$

$$= \frac{E(\mu_A) + E(\nu_A)}{2} = e(A).$$

For the intuitionistic sets A and B, we define their *correlation coefficeint* by the formula

$$\alpha(A, B) = \frac{c(A, B)}{\sqrt{e(A) \cdot e(B)}}. \tag{2.9}$$

Theorem 2.3. $0 \leq \alpha(A, B) \leq 1$, *for any intuitionistic sets A and B.*

Proof. The inequality $\alpha(A, B) \geq 0$ is evident since $c(A, B) \geq 0$ and $e(A)$, $e(B) > 0$. It should be shown that $\alpha(A, B) \leq 1$. We estimate as follows, using by the way the Schwarz inequality and, for simplification, denoting $\mu_{A_i} = \mu_A(x_i)$, $\mu_{B_i} = \mu_B(x_i)$, $\mu_{A_i'} = 1 - \mu_A(x_i)$, etc.:

$$\alpha(A,B) = \frac{c(A,B)}{\sqrt{e(A)\cdot e(B)}} = \frac{\mathrm{C}(\mu_A,\mu_B)+\mathrm{C}(\nu_A,\nu_B)}{2\sqrt{e(A)\cdot e(B)}} =$$

$$= \frac{\sum_{i=1}^{n}(\mu_{A_i}\cdot\mu_{B_i}+\mu_{A_i'}\cdot\mu_{B_i'})+\sum_{i=1}^{n}(\nu_{A_i}\cdot\nu_{B_i}+\nu_{A_i'}\cdot\nu_{B_i'})}{2\left[\frac{\sum_{i=1}^{n}(\mu^2_{A_i}+\mu^2_{A_i'})+\sum_{i=1}^{n}(\nu^2_{A_i}+\nu^2_{A_i'})}{2}\cdot\frac{\sum_{i=1}^{n}(\mu^2_{B_i}+\mu^2_{B_i'})+\sum_{i=1}^{n}(\nu^2_{B_i}+\nu^2_{B_i'})}{2}\right]^{1/2}} \le$$

$$\le \frac{\sum_{i=1}^{n}\mu^2_{A_i}\cdot\sum_{i=1}^{n}\mu^2_{B_i}+\sum_{i=1}^{n}\mu^2_{A_i'}\cdot\sum_{i=1}^{n}\mu^2_{B_i'}+\sum_{i=1}^{n}\nu^2_{A_i}\cdot\sum_{i=1}^{n}\nu^2_{B_i}+\sum_{i=1}^{n}\nu^2_{A_i'}\cdot\sum_{i=1}^{n}\nu^2_{B_i'}}{\left\{\left[\left(\sum_{i=1}^{n}\mu^2_{A_i}+\sum_{i=1}^{n}\mu^2_{A_i'}\right)+\left(\sum_{i=1}^{n}\nu^2_{A_i}+\sum_{i=1}^{n}\nu^2_{A_i'}\right)\right]\left[\left(\sum_{i=1}^{n}\mu^2_{B_i}+\sum_{i=1}^{n}\mu^2_{B_i'}\right)+\left(\sum_{i=1}^{n}\nu^2_{B_i}+\sum_{i=1}^{n}\nu^2_{B_i'}\right)\right]\right\}^{1/2}}.$$

Adopting now the notations

$$\sum_{i=1}^{n}\mu^2_{A_i} = a,\quad \sum_{i=1}^{n}\mu^2_{B_i} = b,\quad \sum_{i=1}^{n}\mu^2_{A_i'} = c,\quad \sum_{i=1}^{n}\mu^2_{B_i'} = d,$$

$$\sum_{i=1}^{n}\nu^2_{A_i} = e,\quad \sum_{i=1}^{n}\nu^2_{B_i} = f,\quad \sum_{i=1}^{n}\nu^2_{A_i'} = g,\quad \sum_{i=1}^{n}\nu^2_{B_i'} = h,$$

we get

$$\alpha(A,B) \le \frac{\sqrt{ab}+\sqrt{cd}+\sqrt{ef}+\sqrt{gh}}{\{[(a+c)+(e+g)]\cdot[(b+d)+(f+h)]\}^{1/2}}.$$

Estimating $\alpha^2(A,B)-1$, one can show that

$$\alpha^2(A,B)-1 \le$$

$$\frac{-[(ad+cb)^2+(af+be)^2+(ah+bg)^2+(cf+dc)^2+(ch+dg)^2+(eh+fg)^2]}{(a+c+e+g)(b+d+f+h)} \le 0,$$

whence $\alpha(A,B) \le 1$. $\Diamond$

Theorem 2.4. *An arithmetical correlation of intuitionistic sets is equal to zero if and only if these sets are Zadeh fuzzy sets.*

The proof of this theorem is immediate on the ground of the construction of formula (2.7) and Prop. 2 from [5].

A further consequence of the above theorem is the fact that such sets must be ordinary sets (non-fuzzy ones).

Definition 2.3. If $c(A,B) = 0$, then the intuitionistic sets A and B are said to be *uncorrelated*.

One can further extend the reasoning described above to three or more intuitionistic sets and to an infinite set E.

3. Logical approach

Note that the entropy $d(f)$ (1.1) is measured by the quantity H for the membership (the function f) and the non-membership (the function $f' = 1 - f$) of a fuzzy set characterized by a function $f\colon H \to [0,1]$. Making use of the above observation, we introduce the following notion.

Definition 3.1. By an *entropy* $D(A)$ of the intuitionistic set A we mean the quantity

$$D(A) = H(\mu_A) + H(\nu_A), \tag{3.1}$$

where H is the function used in (1.1).

The entropy $D(A)$ may also be written down as

$$D(A) = -K \cdot \sum_{i=1}^{n} [\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i)]. \tag{3.2}$$

It can be proved that $D(A)$ satisfies L1 and L3 as well as (1.4). Unfortunately, it cannot be shown in this case that L2 is satisfied, but we have

Theorem 3.1. *$D(A)$ is a valuation, that is,*

$$D(A \cup B) + D(A \cap B) = D(A) + D(B). \tag{3.3}$$

Proof. $D(A \cup B) + D(A \cap B) = H(\mu_{A\cup B}) + H(\nu_{A\cup B}) + H(\mu_{A\cap B}) + H(\nu_{A\cap B}) = H(\mu_{A\cup B}) + H(\mu_{A\cap B}) + H(\nu_{A\cup B}) + H(\nu_{A\cap B}) = H(\mu_A) + H(\mu_B) + H(\nu_A) + H(\nu_B) = H(\mu_A) + H(\nu_A) + H(\mu_B) + H(\nu_B) = D(A) + D(B)$, since H is a valuation (see [10]). $\Diamond$

Definition 3.2. By an *energy* $E(A)$ of the intuitionistic set A we mean the quantity

$$E(A) = p(\mu_A) + p(\nu_A), \tag{3.4}$$

where p is the function used in (1.5).

The energy $E(A)$ may also be written down as

$$E(A) = \sum_{i=1}^{n} [\mu_A^2(x_i) + \nu_A^2(x_i)]. \tag{3.5}$$

It cannot be proved that $E(A)$ satisfies D1, whereas it is not hard to show that $E(A)$ satisfies D2 and D3 when, as the set A^*, we shall take the intuitionistic set defined in Section 2. We also have (1.7) and

Theorem 3.3. *$E(A)$ is a valuation, that is*

$$E(A \cup B) + E(A \cap B) = E(A) + E(B). \tag{3.6}$$

Proof. $E(A \cup B) + E(A \cap B) = p(\mu_{A\cup B}) + p(\nu_{A\cup B}) + p(\mu_{A\cap B}) + +p(\nu_{A\cap B}) = p(\mu_{A\cup B})+p(\mu_{A\cap B})+p(\nu_{A\cup B})+p(\nu_{A\cap B}) = p(\mu_A)+p(\mu_B)+ +p(\nu_A)+p(\nu_B) = p(\mu_A)+p(\nu_A)+p(\mu_B)+p(\nu_B) = E(A)+E(B)$, since p is a valuation (see [4]). ◊

A consequence of such an approach to the problem of a measure of an intuitionistic set is the possibility of defining the correlation of intuitionistic sets in some other way than it was done in Section 2. This problem was discussed in [7]. The correlation of the intuitionistic sets A and B, described in [7] is given by the formula

$$\tilde{c}(A, B) = \sum_{i=1}^{n} [\mu_A(x_i) \cdot \mu_B(x_i) + \nu_A(x_i) \cdot \nu_B(x_i)], \tag{3.7}$$

and the *coefficient of this correlation* is defined as

$$k(A, B) = \frac{\tilde{c}(A, B)}{\sqrt{E(A) \cdot E(B)}}. \tag{3.8}$$

Assume now that the elements $x_1, x_2, \dots, x_n \in E$ may occur in some experiment with probabilities $p_1, p_2, \dots, p_n$, respectively $\left(p_i \geq 0 \; \sum_{i=1}^{n} p_i = 1\right)$, and that some intuitionistic set A is defined on E. Then we introduce the following concepts.

Definition 3.3. By an *entropy* $D^P(A)$ of the intuitionistic set A in the set E *with a defined probability distribution* P we mean the quantity

$$D^P(A) = -K \sum_{i=1}^{n} p_i [\mu_A(x_i) \ln \mu_A(x_i) + \nu_A(x_i) \ln \nu_A(x_i)]. \tag{3.9}$$

Definition 3.4. By an *energy* $E^P(A)$ of the intuitionistic set A in the set E *with a defined probability measure* P we mean the quantity

$$E^P(A) = \sum_{i=1}^{n} p_i^2 \cdot [\mu_A^2(x_i) + \nu_A^2(x_i)]. \tag{3.10}$$

In the case of the lack of fuzziness, formula (3.10) represents Onicescu's informational energy [12].

Definition 3.5. By a *correlation* of the intuitionistic sets A and B in the set E *with defined probability measures* P and Q we mean the quantity

$$\mathrm{C}^P(A,B) = \sum_{i=1}^{n} p_i q_i[\mu_A(x_i)\cdot\mu_B(x_i) + \nu_A(x_i)\cdot\nu_B(x_i)], \tag{3.11}$$

where p_i is the probability of the appearance of the element x_i in the intuitionistic set A, and q_i in the intuitionistic set B $\left(p_i \geq 0,\ q_i \geq 0,\ \sum_{i=1}^{n} p_i = 1,\ \sum_{i=1}^{n} q_i = 1\right)$.
In the case of the lack of fuzziness, formula (3.11) reduces to Onicescu's informational correlation [11].

The concept of a correlation coefficient in this probabilistic approach is self-evident, as well as its extension to a greater number of intuitionistic sets.

Concluding remarks

Each of the approaches presented here has its advantages and disadvantages. A disadvantage of the arithmetical approach is, for instance, the burden of the performance of a great number of arithmetical operations, but its advantage is that it could be shown in a rather simple way that all the properties of the entropy and energy of an ordinary fuzzy set are satisfied for intuitionistic sets. In the logical approach, a defect may be that the quantities proposed for intuitionistic sets fail to satisfy all the properties that we have for fuzzy sets, but when defined for a more general object (a fuzzy set is a special case of an intuitionistic one), they lose a few required merits. However, the logical approach allows one to pass, in a natural way, from notions for intuitionistic sets to the same notions for fuzzy sets.

Let us still observe that, in the case when the intuitionistic sets considered become ordinary fuzzy sets, the proposed quantities reduce to those described in the introduction and in papers [10], [4], [5]. Let us also emphasize that the restriction of the reference set E to finiteness does not influence applications negatively since, in reality, we encounter finite sets (though with a great number of elements) more frequently than infinite ones. However, an extension of our proposals to infinite sets (countable or uncountable) makes no obstacle.

Our suggestions should not be treated as ultimate, either, since the theory of intuitionistic sets itself is not closed yet, and, all the more, the problem of measuring such sets has not been worked out in full yet.

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