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GENERALIZED MAPPINGS BETWEEN FUZZY TOPOLOGICAL SPACES

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Abstract: In a previous paper [8] we introduced and studied the concept of φ-operation on a fuzzy topology τ on a set X. In this paper we introduce the concept of fuzzy $\varphi\psi$-continuous mappings which generalizes most forms of fuzzy continuity. Also we introduce the concept of fuzzy $\varphi\psi$-open (fuzzy $\varphi\psi$-closed) mappings in which fuzzy open (fuzzy closed) and fuzzy homeomorphism, fuzzy θ-open (fuzzy θ-closed) and fuzzy δ-open (fuzzy

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δ - closed) become special cases. Then we introduce the concept of fuzzy $\varphi\psi$ - homeomorphism, generalizing the concepts of fuzzy homeomorphism, fuzzy θ - homeomorphism and fuzzy δ - homeomorphism. Finally, we prove that these concepts are good extensions.

1. Introduction

In order to unify several characterizations and properties of some fuzzy topological concepts and their weaker and stronger forms, in [8] we introduced and studied the concept of an operation φ on a fuzzy topology τ on a set X. Then we introduced the concepts of φ - closure (φ - interior) of fuzzy sets and φ - closed (φ - open) fuzzy sets. We showed that the collection of φ - open fuzzy sets plays a significant role in the context of fuzzy topology in a natural way analogous to that of the φ - open sets in general topology [5, 9].

In this paper, we introduce the concept of fuzzy $\varphi\psi$ - continuous mappings to unify several characterizations and properties of fuzzy continuity, fuzzy θ - continuity, fuzzy δ - continuity, fuzzy weak - continuity, fuzzy strong θ - continuity, fuzzy almost - continuity, fuzzy almost strong θ - continuity, fuzzy super continuity and fuzzy weak θ - continuity. Then we introduce and study the concepts of fuzzy $\varphi\psi$ - open and fuzzy $\varphi\psi$ - closed mappings. After that we introduce the concept of fuzzy $\varphi\psi$ - homeomorphism, generalizing the concepts of fuzzy homeomorphism, fuzzy θ - homeomorphism and fuzzy δ - homeomorphism. Several characterizations of these mappings are investigated. Finally, Lowen's good extension criterion is used to test all concepts mentioned above.

2. Preliminaries

The class of all fuzzy sets in a universe X will denoted by I^X. Fuzzy sets of X will be denoted by Greek letters as μ, ν, η, etc. Crisp subsets of X will be denoted by capital letters as A, B, C, etc. The value of a fuzzy set μ at the element x of X will be denoted by $\mu(x)$. Fuzzy singletons [10] will be denoted by x_ε, y_ν, z_ρ. The class of all fuzzy singletons will be denoted by $S(X)$. Hence $x_\varepsilon \subseteq \mu$ means $\varepsilon \in]0,1]$ and $\varepsilon \leq \mu(x)$. The definitions and results in a fuzzy topological space (fts, for short) due to Chang [2] have already been standardized. For two fuzzy sets μ and ν, we shall write $\mu q \nu$ (resp. $\mu \overline{q} \nu$) to mean that μ is quasi-coincident (resp. not quasi-coincident) with ν [13]. Let $\mu \in I^X$ and $x_\varepsilon \in S(X)$, by $N_Q(x_\varepsilon)$, $\mathrm{int}(\mu)$, $\mathrm{cl}(\mu)$ and $\mathrm{co}(\mu)$, we mean, the family of all open q-neighbourhoods of x_ε, the interior of μ, the closure of μ and the complement of μ.

Proposition 2.1 [8]. *Let $\mu, \nu \in I^X$ and $\{\mu_j : j \in J\} \subseteq I^X$, then:*
(1) $\mu q \nu \Longrightarrow \mu \cap \nu \neq 0$;
(2) $\mu q \nu \Longleftrightarrow (\exists x_\varepsilon \in S(X))(x_\varepsilon \subseteq \mu$ *and* $x_\varepsilon q \nu)$;
(3) $(\forall (x,y) \in X^2)(\forall (\varepsilon, \nu) \in (]0,1])^2)(x \neq y \Longrightarrow x_\varepsilon \overline{q} y_\nu)$;
(4) $x_\varepsilon \overline{q} \mu \Longleftrightarrow x_\varepsilon \subseteq \mathrm{co}(\mu)$;
(5) $\mu \overline{q} \mathrm{co}(\mu)$;
(6) $\mu \subseteq \nu \Longleftrightarrow (\forall x_\varepsilon \in S(X))(x_\varepsilon \subseteq \mu \Longrightarrow x_\varepsilon \subseteq \nu) \Longleftrightarrow (\forall x_\varepsilon \in \in S(X))(x_\varepsilon q \mu \Longrightarrow x_\varepsilon q \nu)$.

Definition 2.2 [4]. For $\mu \in I^X$ we define
(1) $\mu_\alpha = \{x | x \in X$ and $\mu(x) \geq \alpha\}$ as the *weak α-cut* of μ, where $\alpha \in]0,1]$;
(2) $\mu_{\overline{\alpha}} = \{x | x \in X$ and $\mu(x) > \alpha\}$ as the *strong α-cut* of μ, where $\alpha \in [0,1[$.

The strong 0-cut of μ is called the *support* of μ and is denoted as $\mathrm{supp}(\mu)$.

Definition 2.3 [4]. Let (X,T) be an ordinary topological space. The set of all lower semicontinuous functions from (X,T) into the closed unit interval equipped with the usual topology constitutes a fuzzy topology on X that is called the *induced fuzzy topology* associated with (X,T) and is denoted as $(X, \omega(T))$.

Lemma 2.4 [4]. *Let (X,T) be an ordinary topological space, $\mu \in I^X$ and $A \in 2^X$. Then we have:*
(1) $\mu \in \omega(T) \Longleftrightarrow (\forall \alpha \in [0,1[)(\mu_{\overline{\alpha}} \in T)$;
(2) $\mu \in \omega(T)' \Longleftrightarrow (\forall \alpha \in]0,1])(\mu_\alpha \in T')$;
(3) $A \in T \Longleftrightarrow 1_A \in \omega(T)$;
(4) $A \in T' \Longleftrightarrow 1_A \in \omega(T)'$;
(5) $\mathrm{cl}(1_A) = 1_{\mathrm{cl}(A)}$, *where 1_A denotes the characteristic mapping of $A \subseteq X$.*

Definition 2.5 [8]. Let (X,τ) be a fts. A mapping $\varphi : \tau \to I^X$ such that $(\forall \mu \in \tau)(\mu \subseteq \mu^\varphi)$, where μ^φ denotes the value of φ at μ, is called an *operation on τ*. The family of all operations on a fuzzy topology τ on a set X is denoted by $O_{(X,\tau)}$.

Examples 2.6. The mapping $\varphi : \tau \to I^X$ defined by:

(1) $(\forall \mu \in \tau)(\mu^\varphi = \mu)$, is an operation on τ, the so-called *identity operation* $\imath$;
(2) $(\forall \mu \in \tau)(\mu^\varphi = \operatorname{cl}(\mu))$, is an operation on τ, the so-called *closure operation* cl;
(3) $(\forall \mu \in \tau)(\mu^\varphi = \operatorname{int}(\operatorname{cl}(\mu)))$, is an operation on τ, the so-called *interior-closure operation* $\operatorname{int} \circ \operatorname{cl}$.

Definition 2.7 [8]. An operation $\varphi \in O_{(X,\tau)}$ is said to be:
(1) *regular* $\Longleftrightarrow (\forall x_\varepsilon \in S(X))(\forall(\nu,\eta) \in N_Q^2(x_\varepsilon))(\exists \rho \in N_Q(x_\varepsilon))(\rho^\varphi \subseteq \subseteq \nu^\varphi \cap \eta^\varphi)$;
(2) *monotone* $\Longleftrightarrow (\forall(\nu,\eta) \in \tau^2)(\nu \subseteq \eta \Longrightarrow \nu^\varphi \subseteq \eta^\varphi)$.

It follows immediately that every monotone operation is regular, but the converse may not be true [8].

Definition 2.8 [8]. Let (X,τ) be a fts. The mapping $\varphi^\sim : \tau' \to I^X$ is called an *operation on* τ' iff $(\forall \lambda \in \tau')(\lambda \supseteq \lambda^{\varphi^\sim})$, where τ' denotes the family of all closed fuzzy sets of X. The family of all operations on τ' on a set X is denoted by $O_{(X,\tau')}$.

Definition 2.9 [8]. The operations $\varphi \in O_{(X,\tau)}$ and $\varphi^\sim \in O_{(X,\tau')}$ are said to be *dual* iff $(\forall \nu \in \tau)(\operatorname{co}(\nu^\varphi) = (\operatorname{co}(\nu))^{\varphi^\sim})$. Equivalently, φ and $\varphi^\sim$ are dual iff $(\forall \lambda \in \tau')((\operatorname{co}(\lambda))^\varphi = \operatorname{co}(\lambda^{\varphi^\sim}))$.

Definition 2.10 [8]. Let (X,τ) be a fts, $\varphi \in O_{(X,\tau)}$ and $\mu \in I^X$. Then:
(1) the φ-*closure* of μ, denoted by $\operatorname{cl}_\varphi(\mu)$, is given by:

$$x_\varepsilon \subseteq \operatorname{cl}_\varphi(\mu) \Longleftrightarrow (\forall \eta \in N_Q(x_\varepsilon))(\eta^\varphi q \mu);$$

(2) the φ-*interior* of μ, denoted by $\operatorname{int}_\varphi(\mu)$, is given by:

$$x_\varepsilon q \operatorname{int}_\varphi(\mu) \Longleftrightarrow (\exists \eta \in N_Q(x_\varepsilon))(\eta^\varphi \subseteq \mu).$$

Definition 2.11 [8]. Let (X,τ) be a fts, $\varphi \in O_{(X,\tau)}$ and $\mu \in I^X$. Then:
(1) μ is called φ-*closed* $\Longleftrightarrow \operatorname{cl}_\varphi(\mu) = \mu$;
(2) μ is called φ-*open* $\Longleftrightarrow \operatorname{int}_\varphi(\mu) = \mu$;
(3) μ is φ-open iff $\operatorname{co}(\mu)$ is φ-closed.

Theorem 2.12 [8]. *Let (X,τ) be a fts and $\varphi \in O_{(X,\tau)}$. If φ is regular, then the family of all φ-open fuzzy sets forms a fuzzy topology on X and is denoted by τ_φ. Moreover, $\tau_\varphi \subseteq \tau$.*

Definition 2.13 [8] Let (X,τ) be a fts, $\varphi \in O_{(X,\tau)}$ and $\mu \in I^X$. Then μ is called an $\varphi.q$-*neighbourhood* of $x_\varepsilon \Longleftrightarrow (\exists \nu \in N_Q(x_\varepsilon))(\nu^\varphi \subseteq \mu)$.

Theorem 2.14 [8]. $(\forall \mu \in I^X)$ *(μ is φ-open in $(X,\tau) \Longleftrightarrow \mu$ is open in (X,τ^φ)).*

Definition 2.15 [8] A fts (X,τ) is called:

(1) $\varphi.FT_1$ iff for any $x_\varepsilon, y_\nu \in S(X)$ and $x \neq y$, $(\exists \mu \in N_Q(x_\varepsilon))(\exists \eta \in N_Q(y_\nu))(y_\nu \, \overline{q} \mu^\varphi$ and $x_\varepsilon \, \overline{q} \eta^\varphi)$;
(2) $\varphi.FT_2$ or *F - Hausdorff* iff for any $x_\varepsilon, y_\nu \in S(X)$ and $x \neq y$, $(\exists \mu \in N_Q(x_\varepsilon))(\exists \eta \in N_Q(y_\nu))(\mu^\varphi \cap \eta^\varphi = \emptyset)$;
(3) $\varphi.FR_2$ or *R - regular* iff $(\forall x_\varepsilon \in S(X))(\forall \mu \in N_Q(x_\varepsilon))(\exists \eta \in N_Q(x_\varepsilon))$ $(\eta^\varphi \subseteq \mu)$.

Theorem 2.16 [8]. *A fts (X, τ) is $\varphi.FR_2$ iff $\tau = \tau_\varphi$.*

3. Fuzzy $\varphi\psi$ – continuous mappings

In the remainder of this paper, by (X, τ, φ) and (Y, Δ, ψ) we mean (X, τ) and (Y, Δ) are fts's, φ and ψ are operations on τ and Δ respectively.

Definition 3.1. A mapping f from (X, τ, φ) into (Y, Δ, ψ) is called *F.$\varphi\psi$ - continuous* iff $(\forall x_\varepsilon \in S(X))(\forall \eta \in N_Q(f(x_\varepsilon)))(\exists \nu \in N_Q(x_\varepsilon))$ $(f(\nu^\varphi) \subseteq \eta^\psi)$.

Examples 3.2.
(1) For $\varphi = \imath = \psi$, $F.\varphi\psi$ - continuity coincides with F - continuity [2];
(2) for $\varphi = \text{cl} = \psi$, $F.\varphi\psi$ - continuity coincides with $F.\theta$ - continuity [6];
(3) for $\varphi = \text{int} \circ \text{cl} = \psi$, $F.\varphi\psi$ - continuity coincides with $F.\delta$ - continuity [4];
(4) for $\varphi = \imath$ and $\psi = \text{cl}$, $F.\varphi\psi$ - continuity coincides with F.weak - continuity [1];
(5) for $\varphi = \text{cl}$ and $\psi = \imath$, $F.\varphi\psi$ - continuity coincides with F.strong θ - continuity [7];
(6) for $\varphi = \imath$ and $\psi = \text{int} \circ \text{cl}$, $F.\varphi\psi$ - continuity coincides with F.almost continuity [1, 4];
(7) for $\varphi = \text{cl}$ and $\psi = \text{int} \circ \text{cl}$, $F.\varphi\psi$ - continuous is called *F.almost strong θ - continuous*;
(8) for $\varphi = \text{int} \circ \text{cl}$, and $\psi = \imath$ $F.\varphi\psi$ - continuous is called *F.super - continuous*;
(9) for $\varphi = \text{int} \circ \text{cl}$, and $\psi = \text{cl}$, $F.\varphi\psi$ - continuous is called *F.weakly θ - continuous.*

The next theorem characterizes fuzzy $\varphi\psi$ - continuous mappings in terms of the φ - interior (ψ - interior) and φ - closed (ψ - closed) of fuzzy sets.

Theorem 3.3. *For a mapping $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ the following are equivalent:*

(1) *f is F.*$\varphi\psi$ *- continuous;*

(2) $(\forall \eta \in \Delta)(f^{-1}(\eta) \subseteq \operatorname{int}_{\varphi}(f^{-1}(\eta^{\psi})))$;

(3) $(\forall \mu \in I^X)(f(\operatorname{cl}_{\varphi}(\mu)) \subseteq \operatorname{cl}_{\psi}(f(\mu)))$;

(4) $(\forall \eta \in I^Y)(\operatorname{cl}_{\varphi}(f^{-1}(\eta)) \subseteq f^{-1}(\operatorname{cl}_{\psi}(\eta)))$;

(5) $(\forall \eta \in I^Y)(f^{-1}(\operatorname{int}_{\psi}(\eta)) \subseteq \operatorname{int}_{\varphi}(f^{-1}(\eta)))$.

Proof. (1) $\Longrightarrow$ (2): Let $\eta \in \Delta$ and $x_\varepsilon q f^{-1}(\eta)$. Then $f(x_\varepsilon) q \eta$. By (1), $(\exists \nu \in N_Q(x_\varepsilon))(f(\nu^{\varphi}) \subseteq \eta^{\psi})$ and hence $\nu^{\varphi} \subseteq f^{-1}(\eta^{\psi})$ which implies that $x_\varepsilon q \operatorname{int}_{\varphi}(f^{-1}(\eta^{\psi}))$. Thus by Prop. 2.1 (6), we have $f^{-1}(\eta) \subseteq$ $\subseteq \operatorname{int}_{\varphi}(f^{-1}(\eta^{\psi}))$.

(2) $\Longrightarrow$ (3): Let $\mu \in I^X$ and $f(x_\varepsilon) \not\subseteq \operatorname{cl}_{\psi}(f(\mu))$. Then $(\exists \eta \in$ $\in N_Q(f(x_\varepsilon)))(\eta^{\psi} \overline{q} f(\mu))$ and hence $f^{-1}(\eta^{\psi}) \overline{q} \mu$ which implies $\operatorname{int}_{\varphi}(f^{-1}(\eta^{\psi})) \overline{q} \mu$. From $x_\varepsilon q f^{-1}(\eta)$ and by (2) we obtain $(\exists \rho \in N_Q(x_\varepsilon))$ $(\rho^{\varphi} \subseteq f^{-1}(\eta^{\psi}))$. Hence $\rho^{\varphi} \overline{q} \mu$ and so $x_\varepsilon \not\subseteq \operatorname{cl}_{\varphi}(\mu)$ which implies that $f(x_\varepsilon) \not\subseteq f(\operatorname{cl}_{\varphi}(\mu))$. Thus $f(\operatorname{cl}_{\varphi}(\mu)) \subseteq \operatorname{cl}_{\psi}(f(\mu))$.

(3) $\Longrightarrow$ (4): Let $\eta \in I^Y$. From $ff^{-1}(\eta) \subseteq \eta$, we have $\operatorname{cl}_{\psi}(ff^{-1}(\eta)) \subseteq \operatorname{cl}_{\psi}(\eta)$. By (3), we have $f(\operatorname{cl}_{\varphi}(f^{-1}(\eta))) \subseteq$ $\subseteq \operatorname{cl}_{\psi}(ff^{-1}(\eta)) \subseteq \operatorname{cl}_{\psi}(\eta)$. Thus we have $\operatorname{cl}_{\varphi}(f^{-1}(\eta)) \subseteq f^{-1}(\operatorname{cl}_{\psi}(\eta))$.

(4) $\Longrightarrow$ (5): Let $\eta \in I^Y$ and $x_\varepsilon q f^{-1}(\operatorname{int}_{\psi}(\eta))$. Then $x_\varepsilon \not\subseteq$ $\not\subseteq \operatorname{co}(\operatorname{int}_{\psi}(\eta)) = f^{-1}(\operatorname{cl}_{\psi}(\operatorname{co}(\eta)))$. By (4), we have $x_\varepsilon \not\subseteq$ $\not\subseteq \operatorname{cl}_{\varphi}(f^{-1}(\operatorname{co}(\eta))) = \operatorname{co}(\operatorname{int}_{\varphi}(f^{-1}(\eta)))$ and hence $x_\varepsilon q \operatorname{int}_{\varphi}(f^{-1}(\eta))$. Thus, $f^{-1}(\operatorname{int}_{\psi}(\eta)) \subseteq \operatorname{int}_{\varphi}(f^{-1}(\eta))$.

(5) $\Longrightarrow$ (1): Let $x_\varepsilon \in S(X)$ and $\eta \in N_Q(x_\varepsilon)$. From $\eta^{\psi} \overline{q} \operatorname{co}(\eta^{\psi})$, we have $f(x_\varepsilon) \not\subseteq \operatorname{cl}_{\psi}(\operatorname{co}(\eta^{\psi})) = \operatorname{co}(\operatorname{int}_{\psi}(\eta^{\psi}))$ and hence $f(x_\varepsilon) q \operatorname{int}_{\psi}(\eta^{\psi})$ which implies that $x_\varepsilon q f^{-1}(\operatorname{int}_{\psi}(\eta^{\psi}))$. By (5), we have $x_\varepsilon q \operatorname{int}_{\varphi}(f^{-1}(\eta^{\psi}))$ and hence $(\exists \mu \in N_Q(x_\varepsilon))(\mu^{\varphi} \subseteq f^{-1}(\eta^{\psi}))$ and so $f(\mu^{\varphi}) \subseteq \eta^{\psi}$. $\Diamond$

Corollary 3.4. *Let* $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ *be a mapping. If* $(\forall x_\varepsilon \in$ $\in S(X))(\forall \eta \in N_Q(f(x_\varepsilon)))(\exists \mu \in N_Q(x_\varepsilon) \cap \tau_{\varphi})(f(\mu) \subseteq \eta^{\psi})$, *then* f *is F.*$\varphi\psi$ *- continuous.*

Corollary 3.5. *Let* $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ *is an F.*$\varphi\psi$ *- continuous mapping, then the inverse image of each* ψ *- closed (*ψ *- open) fuzzy set is* φ *- closed (*φ *- open).*

The converse need not be true as can be seen from the following example.

Example 3.6. Let $X = \{x, y\}$ and $\mu, \eta, \rho \in I^X$ defined by:

$$\mu = \underline{0.6} \qquad \rho = \underline{0.3}$$
$$\eta(x) = 0.6 \qquad \eta(y) = 0.7,$$

where $\underline{\alpha}$ denotes the constant mapping with value α. Let $\tau = \{X, \emptyset, \mu, \eta, \rho\}$ and $\Delta = \{X, \emptyset, \eta, \rho\}$. Then (X, τ) and (X, Δ) are fts's. Define $\varphi : \tau \to I^X$ and $\psi : \Delta \to I^X$ by:

$$X^\varphi = X \quad \emptyset^\varphi = \emptyset \quad \mu^\varphi = \mu \quad \eta^\varphi = \eta \quad \rho^\varphi = \underline{0.5} \qquad \Big| \qquad X^\psi = X \quad \emptyset^\psi = \emptyset \quad \eta^\psi = \eta \quad \rho^\psi = \underline{0.4}.$$

Clearly φ and ψ are regular operations. Moreover one easily finds: $\tau_\varphi = \{X, \emptyset, \mu, \eta\}$ and $\Delta_\psi = \{X, \emptyset, \eta\}$. Consider the identity mapping $f : (X, \tau, \varphi) \to (X, \Delta, \psi)$. Then the inverse image of each ψ-open is φ-open but f is not $F.\varphi\psi$-continuous. Indeed, for $x_\varepsilon, \varepsilon = 0.8$ and $\rho \in N_Q(f(x_\varepsilon))$ there is no $\nu \in N_Q(x_\varepsilon)$ such that $f(\nu^\varphi) \subseteq \rho^\psi$.

In the following theorem it is shown that ψ-regularity of the codomain space is a sufficient condition to obtain the converse of Cor. 3.5.

Theorem 3.7. *Let $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ be a mapping. If the inverse image of each ψ-open is φ-open and (Y, Δ) is $\psi.FR_2$, then f is $F.\varphi\psi$-continuous.*

Proof. Let $x_\varepsilon \in S(X)$ and $\eta \in N_Q(f(x_\varepsilon))$. From (Y, Δ) is $\psi.FR_2$ and Th. 2.16, we infer $\eta \in \Delta_\psi$. By hypothesis $f^{-1}(\eta) \in \tau_\varphi$ and $x_\varepsilon q f^{-1}(\eta)$ and hence $(\exists \mu \in N_Q(x_\varepsilon))(\mu^\varphi \subseteq f^{-1}(\eta))$ which implies that $f(\mu^\varphi) \subseteq \subseteq \eta \subseteq \eta^\psi$. Thus f is $F.\varphi\psi$-continuous. $\Diamond$

Theorem 3.8. *A mapping $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ is $F.\varphi\psi$-continuous iff $(\forall x_\varepsilon \in S(X))(\forall \lambda_1 \in \Delta'$ and $f(x_\varepsilon) \not\subseteq \lambda_1)(\exists \lambda_2 \in \tau')(x_\varepsilon \not\subseteq \lambda_2)$ and $f(\lambda_2^{\varphi^\sim}) \supseteq \lambda_1^{\psi^\sim}$, where $\varphi^\sim$, $\psi^\sim$ are the dual operations of φ and ψ respectively.*

Proof. Straightforward. $\Diamond$

Theorem 3.9. *The axioms $\varphi.FT_1$ and $\varphi.FT_2$ are inverse invariant under a $F.\varphi\psi$-continuous injective mapping.*

Proof. As example, we prove the $\varphi.FT_2$ inverse invariance. Let f be a $F.\varphi\psi$-continuous mapping from (X, τ, φ) into (Y, Δ, ψ), where (Y, Δ) is $\psi.FT_2$. Let $x_\varepsilon, y_\nu \in S(X)$ with $x \neq y$. Since f is injective, we have $f(x) \neq f(y)$. From (Y, Δ) is $\psi.FT_2$, we obtain $(\exists \eta_1 \in \in N_Q(f(x_\varepsilon)))(\exists \eta_2 \in N_Q(f(y_\nu)))(\eta_1^\psi \cap \eta_2^\psi = \emptyset)$. By $F.\varphi\psi$-continuity of f, $(\exists \mu_1 \in N_Q(x_\varepsilon))(\exists \mu_2 \in N_Q(y_\nu))(f(\mu_1^\varphi) \subseteq \eta_1^\psi$ and $f(\mu_2^\varphi) \subseteq \eta_2^\psi)$.

Hence $f(\mu_1^\varphi) \cap f(\mu_2^\varphi) = \emptyset$ and so $\mu_1^\varphi \cap \mu_2^\varphi = \emptyset$. Thus (X, τ) is $\varphi.FT_2$-fts. $\Diamond$

Theorem 3.10. *The axiom $\varphi.FR_2$ is inverse invariant under a $F.\varphi\psi$-continuous, F-open and injective mapping.*

Proof. Let f be a $F.\varphi\psi$-continuous, F-open and injective mapping from (X, τ, φ) into (Y, Δ, ψ), where (Y, Δ) is $\psi.FR_2$. Let $x_\varepsilon \in S(X)$ and $\mu \in N_Q(x_\varepsilon)$. From f is F-open, we have $f(\mu) \in N_Q(f(x_\varepsilon))$. Since (Y, Δ) is $\psi.FR_2$, we obtain $(\exists \eta \in N_Q(f(x_\varepsilon)))(\eta^\psi \subseteq f(\mu))$. By $F.\varphi\psi$-continuity of f, $(\exists \nu \in N_Q(x_\varepsilon))(f(\nu^\varphi) \subseteq \eta^\psi)$. Hence, $\nu^\varphi = f^{-1}f(\nu^\varphi) \subseteq$ $\subseteq f^{-1}(\eta^\psi) \subseteq f^{-1}f(\mu) = \mu$ (f being injective). Thus, (X, τ) is $\varphi.FR_2$-fts. $\Diamond$

Theorem 3.11. *If $f, g : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ are $F.\varphi\psi$-continuous mappings, φ is regular and (Y, Δ) is $\psi.FT_2$, then the set $\mu = \cup\{x_\varepsilon \mid x_\varepsilon \in$ $\in I^X$ and $f(x_\varepsilon) = g(x_\varepsilon)\}$ is φ-closed in X and if $\operatorname{cl}_\varphi(\mu) = X$ and $(\forall x_\varepsilon \subseteq \mu)(f(x_\varepsilon) = g(x_\varepsilon))$, then $f = g$.*

Proof. For any $x \in X, f(x_\varepsilon) = g(x_\varepsilon)$ iff $f(x) = g(x)$. Hence, if $x_\varepsilon \not\subseteq \mu$, we have $f(x) \neq g(x)$. Since (Y, Δ) is $\psi.FT_2$, then $(\exists \eta_1 \in$ $\in N_Q(f(x_\varepsilon)))(\exists \eta_2 \in N_Q(g(x_\varepsilon)))(\eta_1^\psi \cap \eta_2^\psi = \emptyset)$. By $F.\varphi\psi$-continuity of f and g, $(\exists \nu_1, \nu_2 \in N_Q(x_\varepsilon))(f(\nu_1^\varphi) \subseteq \eta_1^\beta$ and $g(\nu_2^\varphi) \subseteq \eta_2^\beta)$. Then $f(\nu_1^\varphi) \cap g(\nu_2^\varphi) = \emptyset$.

Now, since φ is regular then $(\exists \rho \in N_Q(x_\varepsilon))(\rho^\varphi \subseteq \nu_1^\varphi \cap \nu_2^\varphi)$. In the light of $\eta_1^\psi \cap \eta_2^\psi = \emptyset$, it is easily seen that $\rho^\varphi \cap \mu = \emptyset$ and hence $\rho^\varphi \bar{q} \mu$ which implies that $x_\varepsilon \not\subseteq \operatorname{cl}_\varphi(\mu)$. Thus μ is φ-closed. Finally, since $\mu = \operatorname{cl}_\varphi(\mu) = X$, we have $(\forall x \in X)(\exists x_\varepsilon \subseteq \mu)(f(x_\varepsilon) = g(x_\varepsilon))$ and consequently $(\forall x \in X)(f(x) = g(x))$. Thus $f = g$. $\Diamond$

4. Fuzzy $\varphi\psi$ – open and $\varphi\psi$ – closed mappings

Definition 4.1. A mapping $f : (X, \tau, \varphi) \to (Y, \Delta, \psi)$ is called:

(1) *$F.\varphi\psi$-open* iff for every $\mu \in I^X$, $f(\operatorname{int}_\varphi(\mu)) \subseteq \operatorname{int}_\psi(f(\mu))$;

(2) *$F.\varphi\psi$-closed* iff for every $\mu \in I^X$, $\operatorname{cl}_\psi(f(\mu)) \subseteq f(\operatorname{cl}_\varphi(\mu))$.

Examples 4.2.

(1) If $\varphi = \imath$ and $\psi = \imath$, then $F.\varphi\psi$-open ($F.\varphi\psi$-closed) mapping coincides with F-open (F-closed) [2];

(2) when $\varphi = \operatorname{cl}$ and $\psi = \operatorname{cl}$, then $F.\varphi\psi$-open ($F.\varphi\psi$-closed) mapping is called $F.\theta$-open ($F.\theta$-closed);

(3) if $\varphi = \operatorname{int} \circ \operatorname{cl}$ and $\psi = \operatorname{int} \circ \operatorname{cl}$, then $F.\varphi\psi$-open ($F.\varphi\psi$-closed)

mapping is called $F.\delta$ - open ($F.\delta$ - closed).

Theorem 4.3. *If a mapping $f : (X,\tau,\varphi) \to (Y,\Delta,\psi)$ is $F.\varphi\psi$ - open ($F.\varphi\psi$ - closed), then the image of every φ - open (φ - closed) fuzzy set is ψ - open (ψ - closed). The converse is true if (X,τ) is $\varphi.FR_2$.*

Proof. Let $\mu \in \tau_\varphi$. Then $\mu = \operatorname{int}_\varphi(\mu)$ and hence $f(\mu) = f(\operatorname{int}_\varphi(\mu))$. Since f is $F.\varphi\psi$-open, we have $f(\mu) \subseteq \operatorname{int}_\psi(f(\mu))$ and hence $f(\mu) \in \in \Delta_\psi$. Conversely, if (X,τ) is $\varphi.FR_2$, then by Th. 2.16, we have $(\forall \mu \in \in I^X)(\operatorname{int}_\varphi(\mu) \in \tau_\varphi)$ and hence $f(\operatorname{int}_\varphi(\mu)) \in \Delta_\psi$ which implies that $f(\operatorname{int}_\varphi(\mu)) \subseteq \operatorname{int}_\psi(f(\mu))$. Proof of other case can be given in similar way. $\Diamond$

The next example shows that $\varphi.FR_2$ is needed in the statement Th. 4.3.

Example 4.4. Let $X = \{x,y\}$, $\mu,\nu,\eta,\rho \in I^X$ defined by:

$$\mu(x) = 0.4 \qquad \mu(y) = 0.3 \qquad \eta(x) = 0.7 \qquad \eta(y) = 0.6$$
$$\nu(x) = 0.6 \qquad \nu(y) = 0.7 \qquad \rho = \underline{0.4}$$

Let $\tau = \{X,\emptyset,\mu,\nu\}$ and $\Delta = \{X,\emptyset,\eta,\rho\}$. Then (X,τ) and (X,Δ) are fts's. Define $\varphi : \tau \to I^X$ and $\psi : \Delta \to I^X$ by:

$$X^\varphi = X \qquad \emptyset^\varphi = \emptyset \qquad \Big| \qquad X^\psi = X \qquad \emptyset^\psi = \emptyset$$
$$\nu^\varphi = \nu \qquad \mu^\varphi = \underline{0.4} \qquad \Big| \qquad \eta^\psi = \eta \qquad \rho^\psi = \underline{0.5}\,.$$

Clearly φ and ψ are regular operations. Moreover one easily finds: $\tau_\varphi = = \{X,\emptyset,\nu\}$ and $\Delta_\psi = \{X,\emptyset,\eta\}$ and hence $\tau'_\varphi\{X,\emptyset,\mu\}$ and $\Delta'_\psi = = \{X,\emptyset, \operatorname{co}(\eta)\}$. Define $f : (X,\tau,\varphi) \to (X,\Delta,\psi)$ satisfying $f(x) = y$ and $f(y) = x$, then every image of φ - closed (φ - open) is ψ - closed (ψ - open), but f is not $F.\varphi\psi$ - closed. Indeed, for $\nu \in I^X$, we have $\operatorname{cl}_\varphi(\nu) = = \{(x,0.6)\,,(y,0.9)\}$. So, $f(\operatorname{cl}_\varphi(\nu)) = \{(x,0.9)\,,(y,0.6)\}$. Since $f(\nu) = \eta$, we have $\operatorname{cl}_\psi(f(\nu)) = \operatorname{cl}_\psi(\eta) = \underline{0.9}$. Hence $\operatorname{cl}_\psi(f(\nu)) \nsubseteq \nsubseteq f(\operatorname{cl}_\varphi(\nu))$.

Theorem 4.5. *Let $f : (X,\tau,\varphi) \to (Y,\Delta,\psi)$ be a mapping.*

(1) *If $(\forall \eta \in \tau)(f(\eta) \in \Delta$ and $f(\eta^\varphi) = (f(\eta))^\psi)$, then f is $F.\varphi\psi$ - open.*

(2) *If $(\forall \lambda \in \tau')(f(\lambda) \in \Delta'$ and $f(\lambda^\varphi) = (f(\lambda))^\psi)$, then f is $F.\varphi\psi$ - closed.*

Proof. (1) Let $\mu \in I^X$ and $y_\nu q f(\operatorname{int}_\varphi(\mu))$. Then $(\exists x_\varepsilon \subseteq f^{-1}(y_\nu))$ $(x_\varepsilon q \operatorname{int}_\varphi(\mu))$ and hence $(\exists \eta \in N_Q(x_\varepsilon))(\eta^\varphi \subseteq \mu)$. From hypothesis we obtain that $f(\eta) \in N_Q(y_\nu)$ and $(f(\eta))^\psi \subseteq f(\mu)$ and hence $y_\nu q \operatorname{int}_\psi(f(\mu))$. Thus $f(\operatorname{int}_\varphi(\mu)) \subseteq \operatorname{int}_\psi(f(\mu))$. The proof of (2) is similar. $\Diamond$

Corollary 4.6. *Let $f:(X,\tau,\varphi)\to(Y,\Delta,\psi)$ be a mapping.*

(1) *If $(\forall\eta\in\tau)(f(\eta)\in\Delta$ and $f(\eta^{\varphi})=(f(\eta))^{\psi})$, then the image of every φ - open fuzzy set is ψ - open.*

(2) *If $(\forall\lambda\in\tau')(f(\lambda)\in\Delta'$ and $f(\lambda^{\varphi^{\sim}})=(f(\lambda))^{\psi^{\sim}})$, then the image of every φ - closed fuzzy set is ψ - closed.*

The following example shows that the converse of Cor. 4.6 is not true in general.

Example 4.7. Let $X=\{x,y\}$, $\mu,\nu,\eta,\rho,\sigma\in I^X$ defined by:

$$\mu(x)=0.5 \quad \mu(y)=0.6 \quad \nu(x)=0.8 \quad \nu(y)=0.9$$
$$\eta(x)=0.5 \quad \eta(y)=0.4 \quad \rho(x)=0.4 \quad \rho(y)=0.6$$
$$\sigma=\underline{0.4}\,.$$

Let $\tau=\{X,\emptyset,\mu,\eta,\rho,\sigma\}$ and $\Delta=\{X,\emptyset,\mu,\nu,\rho,\sigma\}$. Then (X,τ) and (X,Δ) are fts's. Define $\varphi:\tau\to I^X$ and $\psi:\Delta\to I^X$ by:

$$X^{\varphi}=X \quad \emptyset^{\varphi}=\emptyset \qquad\Big|\qquad X^{\psi}=X \quad \emptyset^{\psi}=\emptyset$$
$$\mu^{\varphi}=\underline{0.7} \quad \eta^{\varphi}=\eta \qquad\Big|\qquad \mu^{\psi}=\underline{0.6} \quad \nu^{\psi}=\nu$$
$$\rho^{\varphi}=\rho \quad \sigma^{\varphi}=\sigma \qquad\Big|\qquad \rho^{\psi}=\rho \quad \sigma^{\psi}=\sigma\,.$$

It is easy to see that φ and ψ are regular operations and $\tau_{\varphi}=\tau$ and $\Delta_{\psi}=\Delta$. Consider the identity mapping $f:(X,\tau,\varphi)\to(X,\Delta,\psi)$. It is easy to see that the image of every φ - open fuzzy set is ψ - open (and hence f is $F.\varphi\psi$ - open, since (X,τ) is $\varphi.FR_2$), but for $\mu\in\tau$ we have $f(\mu)\in\Delta$ and $f(\mu^{\varphi})\neq(f(\mu))^{\psi}$.

Definition 4.8. A bijective mapping $f:(X,\tau,\varphi)\to(Y,\Delta,\psi)$ is called *$F.\varphi\psi$ - homeomorphism* iff both f and f^{-1} are $F.\varphi\psi$ - continuous.

Example 4.9.

(1) If $\varphi=\imath$ and $\psi=\imath$, then $F.\varphi\psi$ - homeomorphism coincides with F - homeomorphism [2].

(2) If $\varphi=$ cl and $\psi=$ cl, then $F.\varphi\psi$ - homeomorphism is called *$F.\theta$ - homeomorphism.*

(3) If $\varphi=\operatorname{int}\circ\operatorname{cl}$ and $\psi=\operatorname{int}\circ\operatorname{cl}$, then $F.\varphi\psi$ - homeomorphism is called *$F.\delta$ - homeomorphism.*

Theorem 4.10. *If $f:(X,\tau,\varphi)\to(Y,\Delta,\psi)$ is bijective, then the following properties of f are equivalent:*

(1) *f is $F.\varphi\psi$ - homeomorphism;*

(2) *f is $F.\varphi\psi$ - continuous and $F.\varphi\psi$ - open;*

(3) *f is $F.\varphi\psi$ - continuous and $F.\varphi\psi$ - closed;*

(4) $(\forall\mu \in I^X)(f(\operatorname{cl}_{\varphi}(\mu)) = \operatorname{cl}_{\psi}(f(\mu)))$.

Proof. (1) $\Longrightarrow$ (2): Let $\mu \in I^X$. From f^{-1} is $F.\varphi\psi$-continuous, we have $(f^{-1})^{-1}(\operatorname{int}_{\varphi}(\mu)) \subseteq \operatorname{int}_{\psi}((f^{-1})^{-1}(\mu))$ and hence $f(\operatorname{int}_{\varphi}(\mu)) \subseteq$ $\subseteq \operatorname{int}_{\psi}(f(\mu))$.

(2) $\Longrightarrow$ (3): Let $\mu \in I^X$. From f is $F.\varphi\psi$-open and bijective, we obtain that $f(\operatorname{int}_{\varphi}(\operatorname{co}(\mu))) \subseteq \operatorname{int}_{\psi}(f(co(\mu)))$ and hence $\operatorname{co}(f(\operatorname{cl}_{\varphi}(\mu))) \subseteq \operatorname{co}(\operatorname{cl}_{\psi}(f(\mu)))$ which implies that $\operatorname{cl}_{\psi}(f(\mu)) \subseteq$ $\subseteq f(\operatorname{cl}_{\varphi}(\mu))$.

(3) $\Longrightarrow$ (4) and (4) $\Longrightarrow$ (1) can be easily proved. $\Diamond$

5. Good extensions

Definition 5.1 [13]. A property P_f of a fts is said to be a *good extension* of the property P in classical topology iff whenever the fts is topologically generated (induced) say by (X,T), then $(X,\omega(T))$ has property P_f iff (X,T) has property P.

Theorem 5.2 [8]. *Let (X,T) be a topological space and φ be an operation on T. Consider the induced fuzzy topological space $(X,\omega(T))$ and the operation $\varphi_\omega : \omega(T) \to I^X$ defined by: $(\forall\mu \in \omega(T))(\mu^{\varphi_\omega} =$ $= \bigcup_{0\le\alpha<h(\mu)} (\underline{\alpha} \cap 1_{(\mu_{\overline{\alpha}})^{\varphi}}))$, where $h(\mu) = \sup_{x\in X} \mu(x)$. Then:*

(1) $\omega(T_\varphi) = (\omega(T))_{\varphi_\omega}$;

(2) $\operatorname{cl}_{\varphi_\omega}(1_A) = 1_{\operatorname{cl}_{\varphi}(A)}$;

(3) $\operatorname{int}_{\varphi_\omega}(1_A) = 1_{\operatorname{int}_{\varphi}(A)}$;

(4) $\operatorname{cl}_{\varphi_\omega}(\mu) = \bigcup_{0\le\alpha<h(\mu)} (\underline{\alpha} \cap 1_{\operatorname{cl}_{\varphi}(\mu_{\overline{\alpha}})}),\ \forall\mu \in I^X$;

(5) $\operatorname{int}_{\varphi_\omega}(\mu) = \bigcup_{0\le\alpha<h(\mu)} (\underline{\alpha} \cap 1_{\operatorname{int}_{\varphi}(\mu_{\overline{\alpha}})}),\ \forall\mu \in I^X$.

Proposition 5.3. *Let $f : X \to Y$ be a mapping, $\mu \in I^X$, $A \subseteq X$ and $B \subseteq Y$. Then the following relations hold:*

(1) $f^{-1}(\mu_{\overline{\alpha}}) = (f^{-1}(\mu))_{\overline{\alpha}}$.

(2) $f(\mu_{\overline{\alpha}}) = (f(\mu))_{\overline{\alpha}}$.

(3) $f^{-1}(1_B) = 1_{f^{-1}(B)}$.

(4) $f(1_A) = 1_{f(A)}$.

Theorem 5.4. *A mapping $f : (X,T_1,\varphi) \to (Y,T_2,\psi)$ is $\varphi\psi$-continuous iff $f : (X,\omega(T_1),\varphi_\omega) \to (T,\omega(T_2),\psi_\omega)$ is $F.\varphi_\omega\psi_\omega$-continuous.*

Proof. Let $\mu \in (\omega(T_2))_{\psi_\omega}$. From Th. 5.2 (1), we have $\mu \in \omega((T_2)_\psi)$. Then $(\forall\alpha \in [0,1[\,)(\mu_{\overline{\alpha}} \in (T_2)_\psi)$. From f is $\varphi\psi$-continuous and Prop.

5.3 (1), we have $(\forall\alpha \in [0,1[)((f^{-1}(\mu))_{\overline{\alpha}} \in (T_1)_\varphi)$ and hence $f^{-1}(\mu) \in \in \omega((T_1)_\varphi) = (\omega(T_1))_{\varphi_\omega}$. Thus f is $F.\varphi_\omega\psi_\omega$ - continuous. Conversely, let $B \in (T_2)_\psi$. Then by Th. 5.2 (1), $1_B \in (\omega(T_2))_{\psi_\omega}$. Since f is $F.\varphi_\omega\psi_\omega$ - continuous, we have $f^{-1}(1_B) = 1_{f^{-1}(B)} \in \omega((T_1)_\varphi)$ and hence $f^{-1}(B) \in (T_1)_\varphi$. Thus is $\varphi\psi$ - continuous. ◊

Theorem 5.5. *A mapping $f : (X, T_1, \varphi) \to (Y, T_2, \psi)$ is $\varphi\psi$ - open iff $f : (X, \omega(T_1), \varphi_\omega) \to (Y, \omega(T_2), \psi_\omega)$ is $F.\varphi_\omega\psi_\omega$ - open.*

Proof. Let $\mu \in I^X$. Then $(\forall\alpha \in [0,1[)(\mu_{\overline{\alpha}} \subseteq X)$. From f is $\varphi\psi$ - open, it follows $f(\operatorname{int}_\varphi(\mu_{\overline{\alpha}})) \subseteq \operatorname{int}_\psi(f(\mu_{\overline{\alpha}}))$. Then we obtain successively:

$$1_{f(\operatorname{int}_\varphi)(\mu_{\overline{\alpha}})} \subseteq 1_{\operatorname{int}_\psi(f(\mu_{\overline{\alpha}}))}, \quad \underline{\alpha} \cap 1_{f(\operatorname{int}_\varphi(\mu_{\overline{\alpha}}))} \subseteq \underline{\alpha} \cap 1_{\operatorname{int}_\psi(f(\mu_{\overline{\alpha}}))},$$

$$\bigcup_{0\le\alpha<h(\mu)} \left(\underline{\alpha} \cap 1_{f(\operatorname{int}_\varphi(\mu_{\overline{\alpha}}))}\right) \subseteq \bigcup_{0\le\alpha<h(\mu)} \left(\underline{\alpha} \cap 1_{\operatorname{int}_\psi(f(\mu_{\overline{\alpha}}))}\right),$$

$$f\Big(\bigcup_{0\le\alpha<h(\mu)} \left(\underline{\alpha} \cap 1_{\operatorname{int}_\varphi(\mu_{\overline{\alpha}})}\right)\Big) \subseteq \bigcup_{0\le\alpha<h(\mu)} \left(\underline{\alpha} \cap 1_{\operatorname{int}_\psi(f(\mu_{\overline{\alpha}}))}\right).$$

Then $f(\operatorname{int}_{\varphi_\omega}(\mu)) \subseteq \operatorname{int}_{\psi_\omega}(f(\mu))$ and hence f is $F.\varphi_\omega\psi_\omega$ - open.

Conversely, let $A \subseteq X$. Then $1_A \in I^X$ and so $f(\operatorname{int}_{\varphi_\omega}(1_A)) \subseteq \subseteq \operatorname{int}_{\psi_\omega}(f(1_A))$. Then we have successively:

$$f(1_{\operatorname{int}_\varphi(A)}) \subseteq \operatorname{int}_{\psi_\omega}(1_{f(A)}), \quad 1_{f(\operatorname{int}_\varphi(A))} \subseteq 1_{\operatorname{int}_\psi(f(A))}.$$

Then $f(\operatorname{int}_\varphi(A)) \subseteq \operatorname{int}_\psi(f(A))$ and hence f is $\varphi\psi$ - open. ◊

Theorem 5.6. *A mapping $f : (X, T_1, \varphi) \to (Y, T_2, \psi)$ is $\varphi\psi$ - closed iff $f : (X, \omega(T_1), \varphi_\omega) \to (Y, \omega(T_2), \psi_\omega)$ is $F.\varphi_\omega\psi_\omega$ - closed.*

Proof. It is similar to that of Th. 5.5. ◊

With the results seen above we conclude that:

Theorem 5.7. *$f : (X, T_1, \varphi) \to (Y, T_2, \psi)$ is $\varphi\psi$ - homeomorphism iff $f : (X, \omega(T_1), \varphi_\omega) \to (Y, \omega(T_2), \psi_\omega)$ is $F.\varphi_\omega\psi_\omega$ - homeomorphism.*

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