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UNIFIED THEORY FOR SOME SEPARATION AXIOMS

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Abstract: A new class of sets called ψ-generalized closed (briefly ψg-closed) sets in topological spaces are introduced and studied. Some of their properties are investigated here. Finally, some characterizations of ψ-regular and ψ-normal spaces have been given.

1. Introduction

In the past few years, different forms of open sets have been studied. It is observed that a large number of papers is devoted to the study of generalized open like sets of a topological space. The concept of generalized closed sets in a topological space was introduced by N. Levine [13]. After that the concept of generalized closed sets has been investigated by many mathematicians. It is well known that separation axioms are one of the basic subjects of study in general topology and in several branches of mathematics. In very recent papers different mathemati-

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cians [13, 15, 16, 4, 19, 2] continued the study of several weaker forms of separation axioms.

The concept of a generalized type of operator, called operation on the power set $\mathcal{P}(X)$ of a topological space (X, τ), was introduced in [6]. It turns out from the investigations here that by judicious use of the notion of 'operation', one can give generalized definitions of different separation axioms from which the definitions of different varied forms of such properties and many known results thereon follow as particular consequences.

For any topological space (X, τ), the δ-closure [22] of a subset A is defined by $\{x \in X : A \cap U \neq \varnothing$ for all regular open sets U containing $x\}$, where a subset A is called regular open if $A = \text{int}(\text{cl}(A))$. A subset A of a topological space (X, τ) is called preopen [18] (resp. semiopen [14], δ-preopen [21], α-open [17], β-open [1]) if $A \subseteq \text{int}(\text{cl}(A))$ (resp. $A \subseteq \text{cl}(\text{int}(A))$, $A \subseteq \text{int}(\text{cl}_\delta A)$, $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$, $A \subseteq \text{cl}(\text{int}(\text{cl}A))$).

2. Properties of ψg-closed sets

We now begin by recalling a few definitions and observe that many of the existing relevant definitions considered in various papers turn out to be special cases of the ones given below.

Definition 2.1. [6] A mapping $\psi : \mathcal{P}(X) \to \mathcal{P}(X)$ is called an operation on $\mathcal{P}(X)$, where $\mathcal{P}(X)$ denotes as usual the power set of X, if for each $A \in \mathcal{P}(X) \setminus \{\varnothing\}$, $\text{int}A \subseteq \psi(A)$ and $\psi(\varnothing) = \varnothing$.

The set of all operations on a space X will be denoted by $\mathcal{O}(X)$.

Observation 2.2. *It is easy to check that some examples of operations on a space X are the well known operators viz.* int*,* intcl*,* intcl$_\delta$*,* clint*,* intclint*,* clintcl*.*

Definition 2.3. [6] Let ψ denote an operation on a space (X, τ). Then a subset A of X is called ψ-open if $A \subseteq \psi(A)$. Complements of ψ-open sets will be called ψ-closed sets. The family of all ψ-open (resp. ψ-closed) subsets of X is denoted by $\psi\mathcal{O}(X)$ (resp. $\psi\mathcal{C}(X)$).

Observation 2.4. *It is clear that if ψ stands for any of the operators* int*,* intcl*,* intcl$_\delta$*,* clint*,* intclint*,* clintcl*, then ψ-openness of a subset A of X coincides with respectively the openness, preopenness, δ-preopenness, semi-openness, α-openness and β-openness of A* (see [9, 18, 21, 14, 17, 1]).

Definition 2.5. [12] Let (X, τ) be a topological space, $\psi \in \mathcal{O}(X)$ and

$A \subseteq X$. Then the intersection of all ψ-closed sets containing A is called the ψ-closure of A, denoted by ψ-cl A; alternately, ψ-cl A is the smallest ψ-closed set containing A. The union of all ψ-open subsets of G is the ψ-interior of G, denoted by ψ-int$G$.

It is known from [12] that $x \in \psi\text{-cl}\, A$ iff $A \cap U \neq \varnothing$, for all U with $x \in U \in \psi\mathcal{O}(X)$ and $x \in \psi\text{-int}G$ iff $\exists\, x \in U \in \psi\mathcal{O}(X)$ such that $x \in U \subseteq G$. In [12], it is also shown that $X \setminus \psi\text{-cl}\, G = \psi\text{-int}(X \setminus G)$.

Observation 2.6. *Obviously if one takes interior as the operation ψ, then ψ-closure becomes equivalent to the usual closure. Similarly, ψ-closure becomes* $p\,$cl, $p\,\mathrm{cl}_\delta$, $s\,$cl, α-cl, β-cl, *if ψ is taken to stand for the operators* intcl, intcl_δ, clint, intclint *and* clintcl *respectively* (see [9, 18, 21, 14, 17, 1] for details).

Definition 2.7. Let (X, τ) be a topological space and $\psi \in \mathcal{O}(X)$. Then a subset A of X is called a ψ-generalized closed set (or in short, ψg-closed set) iff $\psi\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ where U is ψ-open in X. The complement of a ψg-closed set is called a ψg-open set.

Remark 2.8. (i) In a topological space (X, τ), the definition of g-open set [13] (resp. sg-open set [4, 2], pg-open set [15], $g\alpha$-open set [16]) can be obtained by taking $\psi = \text{int}$ (resp. clint, intcl, intclint).

(ii) Every ψ-open set in a topological space (X, τ) is ψg-open. In fact, if A be a ψ-open set, then $X \setminus A$ is a ψ-closed set. Let $X \setminus A \subseteq U \in \psi\mathcal{O}(X)$. Then $\psi\text{-cl}(X \setminus A) = X \setminus A \subseteq U$. Thus $X \setminus A$ is a ψg-closed set and hence A is a ψg-open set.

The converse of Rem. 2.8(ii) is not true as seen from the next example:

Example 2.9. Let $X = \{a, b, c\}$ and $\tau = \{\varnothing, X, \{a\}, \{a, b\}, \{a, c\}\}$. Then (X, τ) is a topological space. Consider the mapping $\psi : \mathcal{P}(X) \to \to \mathcal{P}(X)$ defined by $\psi(A) = \text{intcl}A$. It is easy to verify that $\{a, c\}$ is ψg-open in (X, τ) but not ψ-open.

The next two examples show that the intersection (union) of two ψg-closed sets is not in general ψg-closed.

Example 2.10. (a) Let $X = \{a, b, c\}$ and $\tau = \{\varnothing, X, \{a\}\}$. Then (X, τ) is a topological space. Consider the mapping $\psi : \mathcal{P}(X) \to \mathcal{P}(X)$ defined by $\psi(A) = \text{int}A$. Then ψ is an operation on the space X and it is easy to see that $\psi\mathcal{O}(X) = \{\varnothing, X, \{a\}\}$. We note that if $A = \{a, c\}$ and $B = \{a, b\}$, then A and B are two ψg-closed sets but $A \cap B = \{a\}$ is not a ψg-closed set.

(b) Let $X = \{a, b, c, d\}$ and $\tau = \{\varnothing, X, \{b\}, \{c, d\}, \{b, c, d\}\}$. Then (X, τ) is a topological space. Consider the mapping $\psi : \mathcal{P}(X) \to \mathcal{P}(X)$ defined by $\psi(A) = \operatorname{intcl} A$. Then ψ is an operation on X. It is easy to check that $\{c\}$ and $\{d\}$ are two ψg-closed sets but their union $\{c, d\}$ is not ψg-closed.

Theorem 2.11. *A subset A of a topological space (X, τ) with an operation ψ is ψg-closed iff ψ-$\mathrm{cl}(A) \setminus A$ contains no non-empty ψ-closed set.*

Proof. Let F be a ψ-closed subset of ψ-$\mathrm{cl}(A) \setminus A$. Then $A \subseteq F^c$ (where F^c denotes as usual the complement of F). Hence by ψg-closedness of A, we have ψ-$\mathrm{cl}(A) \subseteq F^c$, or $F \subseteq (\psi\text{-}\mathrm{cl}(A))^c$. Thus $F \subseteq \psi$-$\mathrm{cl}(A) \cap (\psi\text{-}\mathrm{cl}(A))^c = \varnothing$, i.e., $F = \varnothing$.

Conversely, suppose that $A \subseteq U$ where U is a ψ-open subset of X. If ψ-$\mathrm{cl}(A) \nsubseteq U$, then ψ-$\mathrm{cl}(A) \cap U^c$ $(\neq \varnothing)$ is a ψ-closed subset of ψ-$\mathrm{cl}(A) \setminus A$ – which is a contradiction. Thus ψ-$\mathrm{cl}(A) \subseteq U$. $\Diamond$

Theorem 2.12. *Let A be a ψg-closed subset of a topological space (X, τ) with an operation ψ. Then A is ψ-closed if and only if ψ-$\mathrm{cl}(A) \setminus A$ is ψ-closed.*

Proof. If A is ψ-closed then ψ-$\mathrm{cl}(A) \setminus A = \varnothing$ – which is ψ-closed. Let A be a ψg-closed subset of X such that ψ-$\mathrm{cl}(A) \setminus A$ is a ψ-closed set. Then ψ-$\mathrm{cl}(A) \setminus A$ is a ψ-closed subset of itself. Thus by Th. 2.11, ψ-$\mathrm{cl}(A) \setminus A = \varnothing$ and hence ψ-$\mathrm{cl}(A) = A$, showing A to be a ψ-closed set. $\Diamond$

Theorem 2.13. *Let ψ be an operation on a topological space (X, τ) and A be a ψg-closed set such that $A \subseteq B \subseteq \psi$-$\mathrm{cl}(A)$. Then B is ψg-closed.*

Proof. Let $B \subseteq U$, where $U \in \psi\mathcal{O}(X)$. Since A is ψg-closed and $A \subseteq U$, ψ-$\mathrm{cl}(A) \subseteq U$. Now, $B \subseteq \psi$-$\mathrm{cl}(A) \Rightarrow \psi$-$\mathrm{cl}(B) \subseteq \psi$-$\mathrm{cl}(A)$. So ψ-$\mathrm{cl}(B) \subseteq U$. $\Diamond$

Theorem 2.14. *Let ψ be an operation on a topological space (X, τ). Then $\psi\mathcal{O}(X) = \psi\mathcal{C}(X)$ iff every subset of X is ψg-closed.*

Proof. Suppose $\psi\mathcal{O}(X) = \psi\mathcal{C}(X)$ and A $(\subseteq X)$ be such that $A \subseteq U \in \psi\mathcal{O}(X)$. Then ψ-$\mathrm{cl}(A) \subseteq \psi$-$\mathrm{cl}(U) = U$ and hence A is ψg-closed.

Conversely, suppose that every subset of X is ψg-closed. Let $U \in \psi\mathcal{O}(X)$. Then $U \subseteq U$ and by ψg-closedness of U, we have ψ-$\mathrm{cl}(U) \subseteq U$, i.e., $U \in \psi\mathcal{C}(X)$. Thus $\psi\mathcal{O}(X) \subseteq \psi\mathcal{C}(X)$.

Now, if $F \in \psi\mathcal{C}(X)$ then $F^c \in \psi\mathcal{O}(X)$, so $F^c \in \psi\mathcal{C}(X)$ (as $\psi\mathcal{O}(X) \subseteq \psi\mathcal{C}(X)$), i.e., $F \in \psi\mathcal{O}(X)$. $\Diamond$

Theorem 2.15. *Let ψ be an operation on a topological space (X, τ).*

Then a subset A of X is ψg-open iff $F \subseteq \psi\text{-int}(A)$, whenever F is ψ-closed and $F \subseteq A$.

Proof. Obvious and hence omitted. $\Diamond$

Theorem 2.16. *Let (X, τ) be a topological space and ψ be an operation on X. A subset A is ψg-open iff $U = X$ whenever U is ψ-open and $\psi\text{-int}(A) \cup A^c \subseteq U$.*

Proof. Suppose U is ψ-open and $\psi\text{-int}(A) \cup A^c \subseteq U$. Now, $U^c \subseteq$ $\subseteq (\psi\text{-int}(A))^c \cap A = \psi\text{-cl}(X \setminus A) \setminus (X \setminus A)$. Since U^c is ψ-closed and $X \setminus A$ is ψg-closed, by Th. 2.11, $U^c = \varnothing$, i.e., $U = X$.

Conversely, let F be a ψ-closed set and $F \subseteq A$. Then by Th. 2.15, it is enough to show that $F \subseteq \psi\text{-int}(A)$. Now, $\psi\text{-int}(A) \cup A^c \subseteq$ $\subseteq \psi\text{-int}(A) \cup F^c$, where $\psi\text{-int}(A) \cup F^c$ is ψ-open. Hence by the given condition, $\psi\text{-int}(A) \cup F^c = X$, i.e., $F \subseteq \psi\text{-int}(A)$. $\Diamond$

Theorem 2.17. *Let ψ be an operation on a topological space (X, τ). If a subset A of X is ψg-closed then $\psi\text{-cl}(A) \setminus A$ is ψg-open.*

Proof. Suppose A is ψg-closed and $F \subseteq \psi\text{-cl}(A) \setminus A$, where F is a ψ-closed subset of X. Then by Th. 2.11, $F = \varnothing$ and hence $F \subseteq$ $\subseteq \psi\text{-int}[\psi\text{-cl}(A) \setminus A]$. Then by Th. 2.15, $\psi\text{-cl}(A) \setminus A$ is ψg-open. $\Diamond$

3. Properties of ψ-regular and ψ-normal spaces

Definition 3.1. Let (X, τ) be a topological space and ψ be an operation on the space X. Then (X, τ) is said to be ψ-regular if for each ψ-closed set F of X not containing x, there exist disjoint ψ-open sets U and V such that $x \in U$ and $F \subseteq V$.

Remark 3.2. Regular space, pre-regular space, semi-regular space, β-regular space, α-regular space are defined and studied in [9, 20, 8, 10, 11] respectively. The above definition gives a unified version of all these definitions if ψ takes the role of int, intcl, clint, clintcl, intclint respectively.

Theorem 3.3. *Let ψ be an operation on a topological space (X, τ). Then the following are equivalent:*

(a) *X is ψ-regular.*

(b) *For each $x \in X$ and each $U \in \psi\mathcal{O}(X)$ containing x there exists $V \in \psi\mathcal{O}(X)$ containing x such that $x \in V \subseteq \psi\text{-cl}(V) \subseteq U$.*

(c) *For each ψ-closed set F of X, $\cap\{\psi\text{-cl}(V) : F \subseteq V \in \psi\mathcal{O}(X)\} = F$.*

(d) *For each subset A of X and each $U \in \psi\mathcal{O}(X)$ with $A \cap U \neq \varnothing$, there exists a $V \in \psi\mathcal{O}(X)$ such that $A \cap V \neq \varnothing$ and $\psi\text{-cl}(V) \subseteq U$.*

(e) *For each non-empty subset A of X and each ψ-closed subset F of X with $A \cap F = \varnothing$, there exist $U, V \in \psi\mathcal{O}(X)$ such that $A \cap V \neq \varnothing$, $F \subseteq W$ and $W \cap V = \varnothing$.*

(f) *For each ψ-closed set F with $x \notin F$ there exists $U \in \psi\mathcal{O}(X)$ and a ψg-open set V such that $x \in U$, $F \subseteq V$ and $U \cap V = \varnothing$.*

(g) *For each $A \subseteq X$ and each ψ-closed set F with $A \cap F = \varnothing$ there exists $U \in \psi\mathcal{O}(X)$ and a ψg-open set V such that $A \cap U \neq \varnothing$, $F \subseteq V$ and $U \cap V = \varnothing$.*

(h) *For each ψ-closed set F of X, $F = \cap\{\psi\text{-cl}(V) : F \subseteq V, V$ is ψg-open$\}$.*

Proof. (a) $\Rightarrow$ (b): Let U be a ψ-open set containing x. Then $x \notin X \setminus U$, where $X \setminus U$ is ψ-closed. Then by (a) there exist $G, V \in \psi\mathcal{O}(X)$ such that $X \setminus U \subseteq G$ and $x \in V$ and $G \cap V = \varnothing$. Thus $V \subseteq X \setminus G$ and so $x \in V \subseteq \psi\text{-cl}(V) \subseteq X \setminus G \subseteq U$.

(b) $\Rightarrow$ (c): Let $X \setminus F \in \psi\mathcal{O}(X)$ be such that $x \notin F$. Then by (b) there exists $U \in \psi\mathcal{O}(X)$ containing x such that $x \in U \subseteq \psi\text{-cl}(U) \subseteq X \setminus F$. So, $F \subseteq X \setminus \psi\text{-cl}(U) = V$ (say)$\in \psi\mathcal{O}(X)$ and $U \cap V = \varnothing$. Thus $x \notin \psi\text{-cl}(V)$. Thus $F \supseteq \cap\{\psi\text{-cl}(V) : F \subseteq V \in \psi\mathcal{O}(X)\}$.

(c) $\Rightarrow$ (d): Let $U \in \psi\mathcal{O}(X)$ with $x \in U \cap A$. Then $x \notin X \setminus U$ and hence by (c) there exists a ψ-open set W such that $X \setminus U \subseteq W$ and $x \notin \psi\text{-cl}(W)$. We put $V = X \setminus \psi\text{-cl}(W)$, which is a ψ-open set containing x and hence $A \cap V \neq \varnothing$ (as $x \in A \cap V$). Now $V \subseteq X \setminus W$ and so $\psi\text{-cl}(V) \subseteq X \setminus W \subseteq U$.

(d) $\Rightarrow$ (e): Let F be a ψ-closed set as in the hypothesis of (e). Then $X \setminus F$ is a ψ-open set and $(X \setminus F) \cap A \neq \varnothing$. Then there exists $V \in \psi\mathcal{O}(X)$ such that $A \cap V \neq \varnothing$ and $\psi\text{-cl}(V) \subseteq X \setminus F$. If we put $W = X \setminus \psi\text{-cl}(V)$, then $F \subseteq W$ and $W \cap V = \varnothing$.

(e) $\Rightarrow$ (a): Let F be a ψ-closed set not containing x. Then by (e), there exist $W, V \in \psi\mathcal{O}(X)$ such that $F \subseteq W$ and $x \in V$ and $W \cap V = \varnothing$.

(a) $\Rightarrow$ (f): Obvious as every ψ-open set is ψg-open (by Rem. 2.8(ii)).

(f) $\Rightarrow$ (g): Let F be a ψ-closed set such that $A \cap F = \varnothing$ for any subset A of X. Thus for $a \in A$, $a \notin F$ and hence by (f), there exists $U \in \psi\mathcal{O}(X)$ and a ψg-open set V such that $a \in U$, $F \subseteq V$ and $U \cap V = \varnothing$. So $A \cap U \neq \varnothing$.

(g) $\Rightarrow$ (a): Let $x \notin F$, where F is ψ-closed. Since $\{x\} \cap F = \varnothing$, by (g) there exists $U \in \psi\mathcal{O}(X)$ and a ψg-open set W such that $x \in U$, $F \subseteq W$ and $U \cap W = \varnothing$. Now put $V = \psi\text{-int}(W)$. Then $F \subseteq V$ (by Th. 2.15) and $U \cap V = \varnothing$.

(c) ⇒ (h): We have $F \subseteq \cap\{\psi\text{-cl}(V) : F \subseteq V$ and V is ψg-open$\}$ $\subseteq \cap\{\psi\text{-cl}(V) : F \subseteq V$ and V is ψ-open$\} = F$.

(h) ⇒ (a): Let F be a ψ-closed set in X not containing x. Then by (h) there exists a ψg-open set W such that $F \subseteq W$ and $x \in X \setminus \psi\text{-cl}(W)$. Since F is ψ-closed and W is ψg-open, $F \subseteq \psi\text{-int}(W)$ (by Th. 2.15). Take $V = \psi\text{-int}(W)$. Then $F \subseteq V$, $x \in X \setminus \psi\text{-cl}(V) = U$ (say) (as $(X \setminus F) \cap V = \varnothing$) and $U \cap V = \varnothing$. $\Diamond$

Definition 3.4. Let ψ be an operation on a topological space (X, τ). Then (X, τ) is ψ-normal if for any pair of disjoint ψ-closed subsets A and B of X, there exist disjoint ψ-open sets U and V such that $A \subseteq U$ and $B \subseteq V$.

Remark 3.5. Normal space, pre-normal space, semi-normal space, α-normal space, β-normal space are defined and studied in [9, 20, 2, 11, 10] respectively. The above definition gives a unified version of all these definitions if ψ takes the role of int, intcl, clint, intclint, clintcl respectively.

Theorem 3.6. *Let ψ be an operation on a topological space (X, τ). Then the following are equivalent:*

(a) *X is ψ-normal;*

(b) *For any pair of disjoint ψ-closed sets A and B, there exist disjoint ψg-open sets U and V such that $A \subseteq U$ and $B \subseteq V$;*

(c) *For every ψ-closed set A and ψ-open set B containing A, there exists a ψg-open set U such that $A \subseteq U \subseteq \psi\text{-cl}(U) \subseteq B$;*

(d) *For every ψ-closed set A and every ψg-open set B containing A, there exists a ψ-open set U such that $A \subseteq U \subseteq \psi\text{-cl}(U) \subseteq \psi\text{-int}(B)$;*

(e) *For every ψg-closed set A and every ψ-open set B containing A, there exists a ψ-open set U such that $A \subseteq \psi\text{-cl}(A) \subseteq U \subseteq \psi\text{-cl}(U) \subseteq B$.*

Proof. (a) ⇒ (b): Let A and B be two disjoint ψ-closed subsets of X. Then by ψ-normality of X, there exist disjoint ψ-open sets U and V such that $A \subseteq U$ and $B \subseteq V$. Then U and V are ψg-open by Rem. 2.8(ii).

(b) ⇒ (c): Let A be a ψ-closed set and B be a ψ-open set containing A. Then A and B^c are two disjoint ψ-closed sets in X. Then by (b), there exist disjoint ψg-open sets U and V such that $A \subseteq U$ and $B^c \subseteq V$. Thus $A \subseteq U \subseteq X \setminus V \subseteq B$. Again, since B is ψ-open and $X \setminus V$ is ψg-closed, $\psi\text{-cl}(X \setminus V) \subseteq B$. Hence $A \subseteq U \subseteq \psi\text{-cl}(U) \subseteq B$.

(c) ⇒ (d): Let A be a ψ-closed subset of X and B be a ψg-open set containing A. Since B is a ψg-open set containing A and A is ψ-closed, by Th. 2.15, $A \subseteq \psi\text{-int}(B)$. Thus by (c) there exists a ψg-open set U

such that $A \subseteq U \subseteq \psi$-cl$(U) \subseteq \psi$-int$(B)$.

(d) $\Rightarrow$ (e): Let A be a ψg-closed set and B be a ψ-open set in X containing A. Now $A \subseteq B$ implies ψ-cl$(A) \subseteq B$, where ψ-cl(A) is ψ-closed and B is ψg-open (as B is ψ-open). Then by (d), there exists a ψ-open set U such that $A \subseteq \psi$-cl$(A) \subseteq U \subseteq \psi$-cl$(U) \subseteq \psi$-int$(B)$. Thus $A \subseteq$ $\subseteq \psi$-cl$(A) \subseteq U \subseteq \psi$-cl$(U) \subseteq B$.

(e) $\Rightarrow$ (a): Let A and B be two disjoint ψ-closed subsets of X. Then A is ψg-closed and $A \subseteq X \setminus B$, where $X \setminus B$ is ψ-open. Thus by (e), there exists a ψ-open set U such that $A \subseteq \psi$-cl$(A) \subseteq U \subseteq \psi$-cl$(U) \subseteq X \setminus B$. Thus $A \subseteq U$, $B \subseteq X \setminus \psi$-cl$(U)$ and $U \cap (X \setminus \psi\text{-cl}(U)) = \varnothing$. Hence X is ψ-normal. $\Diamond$

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