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OPTIMAL DISPATCH AND PRICING IN DEREGULATED ELECTRICITY MARKET

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Abstract: The problem of calculating the optimal dispatch and prices in a single-period electricity auction in a wholesale electricity market is considered here. The novel necessary and sufficient conditions of optimality for this problem are derived and computational algorithms for solving these conditions are constructed.

Introduction

Economic dispatch problem for electricity generation has been the focus of numerous studies (e.g. [9], [5]) for a long time. Recent market deregulation led to its reformulation. Calculating market-based economic dispatch now still requires solving of a non-linear programming problem but this problem became qualitatively different. Firstly, its objective

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function became the "cost of generation", defined as suppliers' price-volume bid functions integrated over their dispatched volumes. In most markets these bids are step-wise functions. This yields discontinuities of gradients and reduces the efficiency of standard numerical methods. Secondly, because negative price steps are commonly used in bids the market-based dispatch problem is non-convex and may have multiple solutions. These are the distinguishing characteristics of dispatch problem in a deregulated electricity market which make it important to develop algorithms especially tailored to solve them efficiently.

In this paper we develop novel necessary and sufficient conditions for dispatch and pricing problem in a deregulated electricity market and construct computational methods, based on these conditions, capable of efficiently calculating dispatch and prices.

Dispatch in a deregulated electricity market

We consider normal trading only, did not consider any trading in auxiliary markets, reserves etc. Electricity market is described as a network of scalar flows, which is standard in modeling of economic electricity dispatch. This is not a detailed model of electric currents in a electric network. It should be interpreted as a statistical model of complex system built of electrical currents. It describes electricity market accurate enough to enable its efficient control. The very model considered in this paper is used in practice to dispatch and to price electricity in the Australian market, NEM see [6], [4] and in many other markets.

Thus, we consider electricity market that includes n regional markets connected by a network of interregional connectors, which transfer electricity between them (Fig. 2). Each participant (generator or consumer) is located in one of the regional markets. All participants in one regional market are paid or pay the same regional price for electricity they sell or consume. That is, each regional auction is a single-price auction. As a rule, regional prices in different regions are different. Every trading day is divided into a sequence of identical single-period auctions. A single period auction is in fact a number of linked regional auctions that take place in all regions simultaneously. The results of a single period auction depends on generators' price-volume bids, the state of the market (regional generations and inter-regional flows) before the auction, and the regional demands. These auction results determine regional dispatches – the amounts of electricity sold by every generator in the region, power flows between regions and the regional prices.

Generators submit price-volume bids to the market operator. These bids are step-wise functions that show how much power they are prepared to supply for a particular price (Fig. 1). Price steps P_i^k in price bids could be (and indeed often are in practice) negative. That is, the producer offers to pay a consumer who agrees to buy its electricity. Trading day

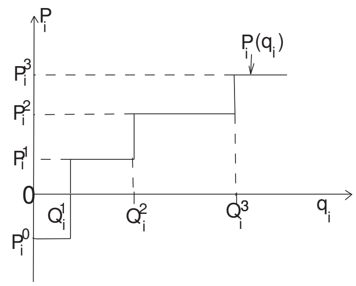

Figure 1: Price-volume bids by producers

is divided into sequence of equal-time periods. Market operator calculates dispatch for each of this period by running single period electricity auction,

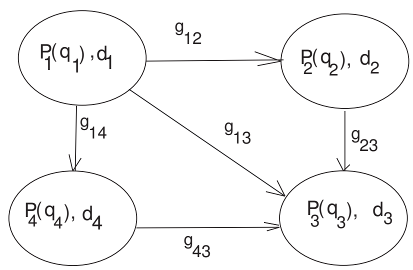

Figure 2: Fragment of regional markets' network

using the following information:

- Combined regional demands by all consumers d_i;
- Combined regional price-bids $P_i(q_i)$ by all generators,
- Combined regional generations before the auction $q_i(0)$,
- Inter-regional flows before the auction $g_{ij}(0)$.

Functions $P_i(q_i)$ are continuous from the right and have left limits $\lim\limits_{q_i \to Q_i^j} P_i(q_i) = P_i^j$. Regional generations are range-constrained $q_i^{\min} \leq q_i \leq q_i^{\max}$.

Inter-regional energy flows g_{ij} from the i-th to j-th region are also range constrained, $g_{ij}^{\min} \leq g_{ij} \leq g_{ij}^{\max}$. These constraints are due to technological and dynamical reasons. Note that $g_{ij} = -g_{ji}$.

We assume that when the power g_{ij} is transferred from i-th to the j-th regional markets some of it is lost and this loss is described by the function $L_{ij}(g_{ij})$. This function is a given continuous function of g_{ij} such that $L_{ij}(g_{ij}) = L_{ji}(-g_{ij}) = L_{ji}(g_{ji})$, $L_{ii}(x) \equiv 0$, $L_{ij}(0) = 0$, $L_{ij} = L_{ji}$, $L_{ij} > 0$ for $g_{ij} \neq 0$ and $\frac{d^2 L_{ij}}{d(g_{ij})^2} > 0$ for $\forall g_{ij}$. Characteristic dependence $L_{ij}(g_{ij})$ on g_{ij} is shown in Fig. 3. The loss L_{ij} is divided

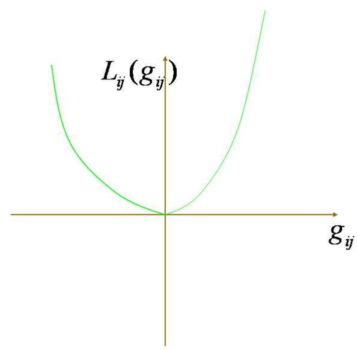

Figure 3: Characteristic dependence of power loss L_{ij} on the power flow g_{ij}

between corresponding markets in a fixed proportion giving additional demand $\alpha_{ij}L_{ij}$ in the i-th region and $\alpha_{ji}L_{ij}$ in the j-th region, Fig. 4. α_{ij} are constants, $0 \leq \alpha_{ij} \leq 1$, $\alpha_{ji} = 1 - \alpha_{ij}$, $\alpha_{ii} = 0$.

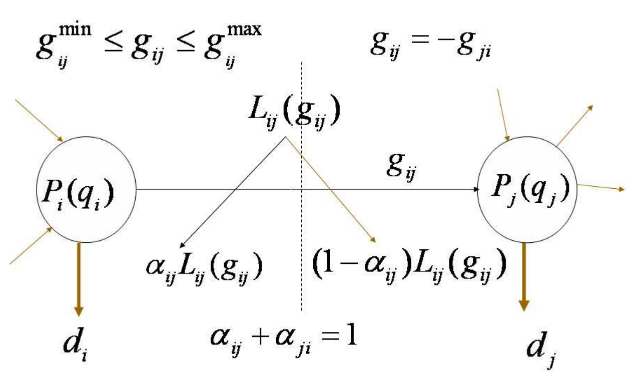

Figure 4: Two markets linked by inter-connector

The net energy balance for the whole network of regional markets is

$$\sum_{i=1}^{n} q_i = \sum_{i=1}^{n} d_i + \frac{1}{2} \sum_{i,j=1, i \neq j}^{n} L_{ij}(g_{ij}). \tag{1}$$

Since $g_{ij} = -g_{ji}$ and $\alpha_{ij} + \alpha_{ji} = 1$, $\forall i, j = 1, \ldots, n$ this balance holds if the regional energy balances

$$q_i = d_i + \sum_{j=1, j \neq i}^{n} \big(g_{ij} + \alpha_{ij} L_{ij}(g_{ij})\big), \quad i = 1, \ldots, n \tag{2}$$

hold. The cost of generation in the i-th market is defined as

$$C_i(q_i) = \int_0^{q_i} P_i(x) dx. \tag{3}$$

The optimal dispatch is determined by minimizing the combined cost of generation of all generators in all regional markets, I, on q_i, g_{ij}

$$I(d_1, \ldots, d_n, q_i, \ldots, q_n, g_{ij}) = \sum_{i=1}^{n} C_i(q_i) \to \min_{q_i, g_{ij}} \tag{4}$$

subject to range constraints

$$q_i^{\min} \leq q_i \leq q_i^{\max}, \quad i = 1, \ldots, n \tag{5}$$

$$g_{ij}^{\min} \leq g_{ij} \leq g_{ij}^{\max} \quad j = 1, \ldots, n, \qquad i = 1, \ldots, n \tag{6}$$

and regional markets' energy balances (2). These balances include non-linear functions $L_{ij}(g_{ij})$ which makes the dispatch problem non convex. We denote its solution as q_i^*, g_{ij}^* and the value of the objective function (the minimal cost of supply) as

$$\begin{aligned} I^*(d_1, \ldots, d_n) &= \min_{q_i, g_{ij}} I(d_1, \ldots, d_n, q_i, \ldots, q_n, g_{ij}) = \\ &= I(d_1, \ldots, d_n, q_i^*, \ldots, q_n^*, g_{ij}^*). \end{aligned} \tag{7}$$

The cost of generation I and the minimal cost of generation I^* both depend not only on d_i but also on the bids $P_i(q_i)$ and pre-auction state of the market $q_i(0)$, $g_{ij}(0)$ (via range constraints on q_i and g_{ij}).

The unknown variables in the optimal dispatch problem (4), (5), (6), (2) are the regional generations and inter-regional flows after the auction. If network contains connections between all regional markets then the number of unknown variables here is $\frac{n^2-n}{2}+n=\frac{n(n+1)}{2},\ n\geq 0$. The first term is the number of unknown exchange flows g_{ij} and the second term is the number of generated regional powers q_i. Since these variables obey n balance equations (2) the number of free unknowns in the dispatch problem is equal to the number of inter-regional flows, $\frac{n(n-1)}{2}$ for a network where all regional markets are connected. For $n=1$ there is one unknown, for $n=3$ – three, etc.

Note that the problem (4–6) is not easy to solve, it has both range constraints on its independent variables and constraints on its dependent variables q_i.

Necessary conditions of optimality

The optimal dispatch problem (4), (5), (2) can be rewritten as a non-linear programming problem

$$\sum_{i=1}^{n} C_i\left[d_i+\sum_{j=1,i\neq j}^{n}\big(g_{ij}+\alpha_{ij}L_{ij}(g_{ij})\big)\right]\to\max_{g_{ij}} \tag{8}$$

subject to constraints

$$q_i^{\min}\leq d_i+\sum_{j=1,i\neq j}^{n}\big(g_{ij}+\alpha_{ij}L_{ij}(g_{ij})\big)\leq q_i^{\max} \tag{9}$$

and range constraints

$$g_{ij}^{\min}\leq g_{ij}\leq g_{ij}^{\max}\quad i,j=1,\ldots,n. \tag{10}$$

The independent unknown variables now are the flows g_{ij}. Regional generations are calculated from balances (2).

We define *reduced price* of the region i with respect to region j as

$$\tilde{P}_{ij}(g_{ij})=P_i\big(d_i+\sum_j\big(g_{ij}+\alpha_{ij}L_{ij}(g_{ij})\big)\big)\left[1+\alpha_{ij}\frac{dL_{ij}(g_{ij})}{dg_{ij}}\right], \tag{11}$$

and for j-th with respect to i-th it is

$$\tilde{P}_{ji}(g_{ij}) = P_j\big(d_j + \sum_i \big(-g_{ij} + \alpha_{ji} L_{ij}(g_{ij})\big)\big)\left[1 - \alpha_{ji}\frac{dL_{ij}(g_{ij})}{dg_{ij}}\right].$$

We also define their left and right limits

$$\tilde{P}^{+}_{ij} = \lim_{\epsilon\to 0} \tilde{P}_{ij}(g_{ij}+\epsilon), \quad \tilde{P}^{-}_{ij} = \lim_{\epsilon\to 0} \tilde{P}_{ij}(g_{ij}-\epsilon).$$

The necessary conditions of optimality for dispatch problem is then given by the following

Theorem. *If $\{g^{*}_{ij}\}$ is an optimal solution of the problem* (8)–(10) *then one of the following conditions hold:*

• *range constraints are non-binding and reduced prices are equalized in directly connected regions*

$$\begin{gathered} g^{\min}_{ij} < g^{\star}_{ij} < g^{\max}_{ij}, \\ q^{\min}_{i} < d_i + \textstyle\sum_j \big(g^{\star}_{ij} + \alpha_{ij} L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{i}, \\ q^{\min}_{j} < d_j + \textstyle\sum_i \big(-g^{\star}_{ij} + (1-\alpha_{ij}) L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{j}, \\ \tilde{P}^{+}_{ij}(g^{\star}_{ij}) \geq \tilde{P}^{-}_{ji}(g^{\star}_{ji}), \qquad \tilde{P}^{-}_{ij}(g^{\star}_{ij}) \leq \tilde{P}^{+}_{ji}(g^{\star}_{ji}); \end{gathered} \tag{12}$$

• *flow constraints are binding preventing flow increase/reduction to equalize reduced prices*

$$\begin{gathered} g^{\star}_{ij} = g^{\max}_{ij}, \\ q^{\min}_{i} < d_i + \textstyle\sum_j \big(g^{\star}_{ij} + \alpha_{ij} L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{i}, \\ q^{\min}_{j} < d_j + \textstyle\sum_i \big(-g^{\star}_{ij} + (1-\alpha_{ij}) L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{j}, \\ \tilde{P}^{-}_{ij}(g^{\star}_{ij}) \leq \tilde{P}^{+}_{ji}(g^{\star}_{ji}), \end{gathered} \tag{13}$$

or

$$\begin{gathered} g^{\star}_{ij} = g^{\min}_{ij}, \\ q^{\min}_{i} < d_i + \textstyle\sum_j \big(g^{\star}_{ij} + \alpha_{ij} L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{i}, \\ q^{\min}_{j} < d_j + \textstyle\sum_i \big(-g^{\star}_{ij} + (1-\alpha_{ij}) L_{ij}(g^{\star}_{ij})\big) < q^{\max}_{j}, \\ \tilde{P}^{+}_{ij}(g^{\star}_{ij}) \geq \tilde{P}^{-}_{ji}(g^{\star}_{ji}); \end{gathered} \tag{14}$$

• *generation constraint is binding* (A) *preventing reduced prices equalization*

(15)
$$g_{ij}^{\min} < g_{ij}^\star < g_{ij}^{\max}, \qquad \tilde{P}_{ij}^- \le \tilde{P}_{ji}^+,$$
$$q_i^{\min} = d_i + \sum_j \big(g_{ij}^\star + \alpha_{ij} L_{ij}(g_{ij}^\star)\big),$$
$$or$$
$$q_j^{\max} = d_j + \sum_j \big(-g_{ij}^\star + (1-\alpha_{ij}) L_{ij}(g_{ij}^\star)\big),$$

or (B)

(16)
$$g_{ij}^{\min} < g_{ij}^\star < g_{ij}^{\max}, \qquad \tilde{P}_{ij}^+ \ge \tilde{P}_{ji}^-,$$
$$q_i^{\max} = d_i + \sum_j \big(g_{ij}^\star + \alpha_{ij} L_{ij}(g_{ij}^\star)\big),$$
$$\text{or}$$
$$q_j^{\min} = d_j + \sum_j \big(-g_{ij}^\star + (1-\alpha_{ij}) L_{ij}(g_{ij}^\star)\big).$$

Proof. First we consider the point where both range constraints (9), (10) are not binding and P_i and P_j are continuous on g_{ij}. Here the objective function I has the same directional derivative on g_{ij} in any direction and the necessary condition of optimality is reduced to the condition

$$\frac{\partial I}{\partial g_{ij}} = (\tilde{P}_{ij} - \tilde{P}_{ji}) = 0.$$

Since arguments of the reduced prices here are at the points where the bid functions are continuous, $\tilde{P}_{ij}^+ = \tilde{P}_{ij}^- = \tilde{P}_{ij}$ and $\tilde{P}_{ji}^+ = \tilde{P}_{ji}^- = \tilde{P}_{ji}$, and this condition coincides with (12).

Let us consider the general case of possible discontinuous points of P_i and/or P_j. Suppose that all range constraints are not binding. Check for an optimality then is reduced to checking that a scalar function I of a scalar parameter g_{ij} has minimum at a point g_{ij}^* where g_{ij}^* can be either increased or decreased. The directional derivatives ([8]) of I along two feasible directions $\overline{g_{ij}^+} = \{0, \dots, \frac{g_{ij}}{|g_ij|}, \ , \dots, 0\}$ and $\overline{g_{ij}^-} = \{0, \dots, -\frac{g_{ij}}{|g_ij|}, \dots, 0\}$ are

(17)
$$\Delta I_{g_{ij}^+} = (P_i^+ - P_j^-)$$

(18)
$$\Delta I_{g_{ij}^-} = (P_j^+ - P_i^-)$$

correspondingly. Then the conditions of optimality for I have the form ([8])

$$\Delta I_{g_{ij}^+} = (P_i^+ - P_j^-) = 0, \tag{19}$$

$$\Delta I_{g_{ij}^-} = (P_j^+ - P_i^-) = 0 \tag{20}$$

which coincides with (12).

Finally, if at least one of the following conditions $g_{ij} = g_{ij}^{\max}$ or $q_i^{\max} = d_i + \sum_j \big(g_{ij} + \alpha_{ij} L_{ij}(g_{ij})\big)$ holds, then it is not feasible to increase g_{ij}^*. In this case the necessary condition of optimality includes only the second equation (20) and the binding range constraint, see (16). The same derivations for the case, where it is only feasible to decrease g_{ij}^* because of the binding range constraint concludes the proof.

Consider a linear subnetwork of three regional markets, Fig. 5. Suppose that

$$\tilde{P}_{ij} < \tilde{P}_{ji}, \qquad \tilde{P}_{jk} < \tilde{P}_{kj}, \qquad q_j^* = q_j^{\max}.$$

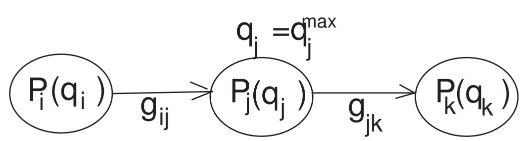

Figure 5: 3-market linear fragment

The flow is directed from i-th to the j-th and then j-th to the k-th markets. The range constraint on the generation in the j-th market is active. This prohibits unconditional increase of g_{ij}. However, two simultaneous equal increases of g_{ij} and g_{jk} are feasible. Analysis identical to a single connector analysis above yields the following conditions of optimality for non-binding range constraints for q_i and q_k on the intervals where $P_i(.)$ and $P_k(.)$ are continuous

$$P_i\big(1 - \alpha_{ij} L'_{ij}\big)\big(1 - \alpha_{jk} L'_{jk}\big) = P_k\big(1 + \alpha_{ij} L'_{ij}\big)\big(1 + \alpha_{jk} L'_{jk}\big), \tag{21}$$

and the general condition of optimality for discontinuous points

$$P_i^+\big(1 - \alpha_{ij} L'_{ij}\big)\big(1 - \alpha_{jk} L'_{jk}\big) \le P_k^-\big(1 + \alpha_{ij} L'_{ij}\big)\big(1 + \alpha_{jk} L'_{jk}\big), \tag{22}$$

$$P_i^-\big(1 - \alpha_{ij} L'_{ij}\big)\big(1 - \alpha_{jk} L'_{jk}\big) \ge P_k^+\big(1 + \alpha_{ij} L'_{ij}\big)\big(1 + \alpha_{jk} L'_{jk}\big). \tag{23}$$

For binding range constraints these condition of long-range optimality is exactly the same as for short-range optimality above – but for long-range reduced prices (lhs and rhs of (21)) instead of short-range reduced prices (11). They state that for the optimal dispatch two regional markets connected by a network of generation-constrained intermediate markets are different, can have different long-range reduced prices if and only if at least one of the flows between them is constraint or the generation in one these two markets is range constrained.

Numerical algorithm

The derived conditions of optimality require that reduced prices between linked regional markets be equal unless flow constraint or generation constraint becomes binding. They can be used to construct algorithm for dispatch optimization.

As an initial point this algorithm uses any feasible set of flows g_{ij} such that range constraints (6) and (9) hold. This solution is then improved iteratively by applying to it the following elementary operation:

(1) calculate the reduced prices difference $\Delta\tilde{P}_{ij} = \tilde{P}_{ij} - \tilde{P}_{ji}$ for every pair of regions connected by inter-connector;

(2) mark all pair where one of the optimality conditions holds;

(3) for each unmarked pair if $\tilde{P}^{-}_{ij} > \tilde{P}^{+}_{ji}$ then $g^{\min}_{ij} < g_{ij} < g^{\max}_{ij}$, $q^{\max}_{i} < q_i(g_{ij}) < q^{\max}_{i}$, $q^{\max}_{j} < q_j(g_{ij}) < q^{\max}_{j}$ then g_{ij} is increased, until one of these inequalities becomes equality. This increase continues across jumps of the price-volume bids, provided that the reduced i-th price at these new steps is still lower that reduced price $\tilde{P}_{ji}$. Similarly, if $\tilde{P}^{-}_{ij} < \tilde{P}^{+}_{ji}$ then g_{ij} is reduced until either reduced prices equalize or one of range constraints becomes binding. The improved solution is again submitted as an input to step (2).

Each iteration reduces the total cost of generation because it reduces the fraction of this cost contributed by the i and j markets (from the proof of the Theorem above it follows that this fraction can always be improved if reduced prices in i-th and j-th markets differ and if range constraints are not binding) and leaves the rest of the cost of generation unchanged. After iterations that use all two connected markets converge, one can use its output as an initial solution for another iterative algorithm which uses the improvement operation for 3-market substrings instead of 2-market ones, then for 4-market etc. Algorithm will stop only when it converges to the solution which obeys the necessary conditions of optimality given by the above Theorem.

Note that it is not necessary to increase / decrease g_{ij} during stage (2) of the algorithm monotonically. Since this is a discrete search, and an extremum can be located within one step in three points only, any discrete search methods for a sorted array can be used here. This algorithm is not a gradient type algorithm. It solves the optimal dispatch problem by solving its necessary conditions of optimality derived above directly.

In practice market operator calculates dispatch and prices by linearizing the above-described dispatch problem and then by solving it numerically using classical linear programming algorithms (see [6], [4]).

The conditions of optimality, are reduced to the equalization of reduced prices between all regions across the network. It can be used to design an automatic continuous control system which would replace the sequence of single-auctions with one continuous real-time auction. These would allow to dramatically improve dispatch.

Non-convexity of the dispatch problem when bids have negative price steps

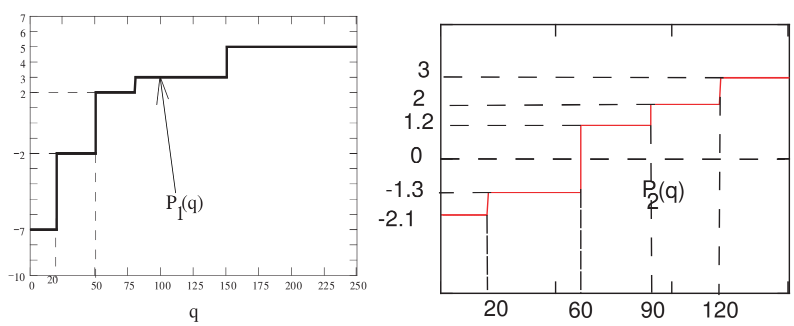

Figure 6: Bids by generators in two regional markets

We illustrate the non-convexity of the dispatch problem where negative price steps are used we plot the cost of generation $I(g)$ as a function of g in Fig. 7 for the market with two regional markets, regional bids $P_1(q)$ and $P_2(q)$ shown in Fig. 6, and losses $L(g) = 0.14g^2$ equally divided between regional markets. The graph clearly shows that $I(g)$ has two minima. The left minimum corresponds to $I^* = -269.137$, $g^* = -11.21$ and dispatches $q_1^* = 37.59$ and $q_2^* = 60$.

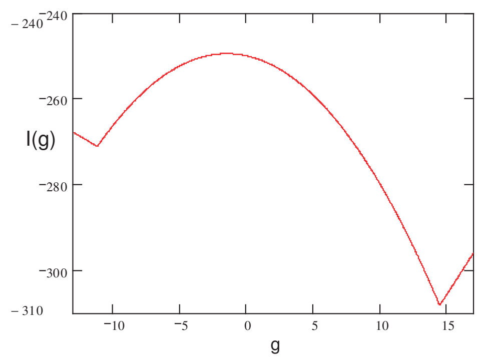

Figure 7: Cost of supply for two regional markets in the example as the function of inter-regional flow

The link between non-convexity and negative prices in bids can be established using the following analysis. Each regional cost of generation (integrated price-volume bids) $C_i(q_i)$ is a convex, piece-wise linear function. Therefore, the total cost of generation – the sum of regional costs of generation – is also convex on q_i. But after we express these variables in terms of inter-regional flows g_{ij} using energy balances (2) the objective function of the single-period auction problem becomes non-convex. Indeed, the second derivative of $C_i(g_{ij})$ with respect to g_{ij} is

$$\frac{d^2}{dg_{ij}^2}C_i = \frac{d}{dg_{ij}}\left(\frac{dC_i}{dq_i}\frac{dL_{ij}}{dg_{ij}}\right) = \sum_j P_i^* \frac{dL_{ij}}{dg_{ij}^2}\,. \tag{24}$$

Since $\frac{dL_{ij}}{dg_{ij}^2} > 0$ the sign of $\frac{d^2}{dg_{ij}^2}C_i$ coincides with the sign of the price-volume bid (current price step). Thus, the use of negative price steps in bids is the causes of non-convexity.

Thus, if one solves the necessary conditions of optimality derived above or uses a direct search to minimize the cost of supply, then there is no guarantee the the solution found is global minimum, only that it is one of the local minima. That is why it is important to verify if the solution found is local or global minimum and it is local then how close the local minimum is from the global one, how much improvement in terms of the cost of supply can be achieved if search for optimal dispatch continues from another initial point.

Globally bound on the optimal dispatch

Consider relaxation ([2]) of the optimal dispatch problem (4), (5), (6), (2) by deleting range constraints on the flows (6) and regional energy balances (2). The generalized optimal dispatch problem then becomes

$$I(d_1,\dots,d_n,q_i,\dots,q_n,g_{ij})) = \sum_{i=1}^{n} C_i(q_i) \to \min_{q_i} \tag{25}$$

subject to

$$\sum_i q_i = M \tag{26}$$

$$q_i^{\min} \le q_i \le q_i^{\max}, \quad i = 1,\dots,n. \tag{27}$$

M here is the parameter that describes inter-connectors losses and which must obey the range constraints

$$\sum_i Q_i^{\min} \le \sum_i d_i \le M \le \sum_i q_j^{\max}. \tag{28}$$

Bellman function ([1]) is defined using the following recurrent equation

$$\phi_1(x_1) = C_1(x_1), \quad \phi_2(x_2) = \min_{q_2^{\min}\le q_2\le q_2^{\max}} \big[C_2(q_2) + \phi_1(x_2 - q_2)\big],$$

$$\phi_\nu(x_\nu) = \min_{q_\nu^{\min}\le q_\nu\le q_\nu^{\max}} \big[C_\nu(q_\nu) + \phi_\nu(x_\nu - q_\nu)\big]. \tag{29}$$

By construction

$$\phi_n(M) = \min_{q_1,q_2,\dots,q_n} \sum_i C_i(q_i^*(M)) \tag{30}$$

gives global minimum and corresponding network losses are given by the network energy balance

$$\sum_i d_i + \frac{1}{2}\sum_{ij} L_{ij}(g_{ij}) = M. \tag{31}$$

Suppose we applied the algorithm described above and obtained g_{ij}^*, q_i^*, I^* and $M^* = \sum_i q_i^*$. Then $\phi_h(M^*)$ will give the lower bound on the cost of generation I. If it turns out that $I^* = \phi_n(M^*)$ then g_{ij}^*, q_i^* is the global minimum. If $|I^* - \phi_n(M^*)|$ is small enough then one may reasonably decide to stop search and use the current local minimum instead of global one.

Entrepreneurial inter-connectors

Entrepreneurial lines ([7]) represent another new feature of modern electricity markets. Physically they are no different to the standard inter-regional connectors that are controlled by market operator. But entrepreneurial lines operate differently. Before trading starts they submit price-volume bids, that are identical to generators price-volume bids. They submits these bids for two regional markets connected by their line. These bids are the offers to flow energy, which are taken into account when single-period auction is calculated.

Let us consider for simplicity two regional market i-th and j-th linked by an entrepreneurial line, see Fig. 8. We denote their price-volume bids as $E_i(x)$ and $E_j(x)$, the flow and losses in the entrepreneurial line as e and $LE(e)$, the coefficient that shows how these losses are apportioned to the i-th and j-th regions as α_e and the contribution of the line to the net energy balances in the i-th and j-th regions (amount flown in/out of the region) as qe_i and qe_j. The dispatch is then found by solving the extended optimal dispatch problem (4), (5), (6), (2),

$$I(d_1,\dots,d_n) = C_i(q_i) + C_j(x_j) + \frac{\operatorname{sign}(e)+1}{2}\int_0^{qe_i} E_i(x)dx - \\ - \frac{\operatorname{sign}(e)-1}{2}\int_0^{qe_j} E_j(x)dx \to \min_{q_i,g_{ij},qe_i,qe_j,e} \tag{32}$$

subject to regional energy balances

$$q_i = d_i - \sum_j (g_{ij} - \alpha_{ij}L_{ij}(g_{ij})) + e + \alpha_e LE(e), \tag{33}$$

range constraints (6), and

$$e^{\min} \le e \le e^{\max} \tag{34}$$

furthermore balances for the entrepreneurial line

$$qe_i = e + \alpha_e LE(e), \quad qe_j = -e + (1-\alpha_e)LE(e), \quad qe_i + qe_j = LE(e). \tag{35}$$

In (32) $\operatorname{sign}(s) = 1$ if $s \ge 0$ and $\operatorname{sign}(x) = -1$ if $x < 0$. This reflects the nature of operations by entrepreneurial line – it flows energy from one regional market to another. The cost of generation now includes the cost of supply for entrepreneurial line itself. If flow is directed from i to j then the extra term corresponding to this cost is $C(qe_i) = \int_0^{qe_i} E_i(x)dx$, if the energy is transferred from j to i then this added term is $C(qe_j) = = \int_0^{qe_j} E_j(x)dx$. For the positive e the variation of I is

$$\delta I = \left[P_i \Big(1 + \alpha_e \frac{d}{de}(LE)\Big) + P_j \Big(-1 + (1 - \alpha_e)\frac{d}{de}(LE)\Big) + + E_i(e) \Big(1 + \alpha_e \frac{d}{de}(LE)\Big) \right] \delta e. \tag{36}$$

If e belongs to an interval where function $E_i(.)$ is continuous, then the following necessary condition of optimality take the form

$$E_i^* = (P_j^* - P_i^*) + \frac{\frac{d}{de}(LE)}{1 + \alpha_e \frac{d}{de}(LE)} P_j^*. \tag{37}$$

That is, the highest price band dispatched via the entrepreneurial line

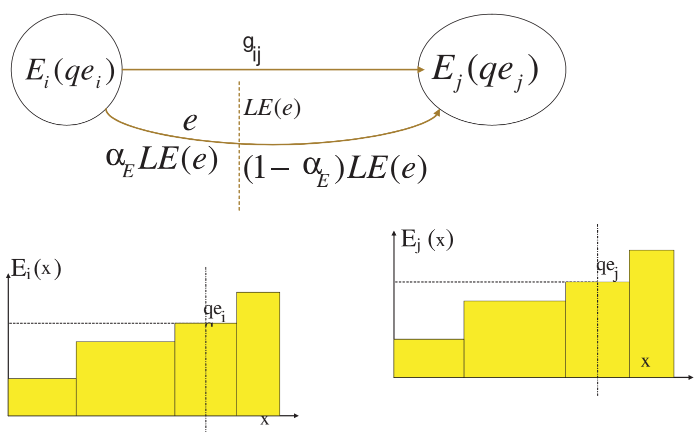

Figure 8: Entrepreneurial inter-connectors

is determined by the difference between spot prices in these two markets plus correctional term that depends on the network losses in entrepreneurial line.

In algorithmic terms if

$$P_i(1 + \alpha_e LE') + P_j(-1 + (1 - \alpha_e)LE') < E_i(e)(1 + \alpha_e LE') \tag{38}$$

then one needs to increase e until this condition is no longer true.

Conclusions

The dispatch and pricing problem for single-period electricity auction in a network of regional markets is considered. Its conditions of optimality are derived. It is shown these conditions simply require that the values of some well-defined function, called reduce price, in connected regional markets be as close to each other as possible. Computational algorithms for solving these conditions numerically are constructed. It is shown that dispatch problem is non-convex if negative prices are used in bids. The dynamic programming based algorithm for calculation of the lower bound on the cost of generation is constructed. It is shown how it can be used to verify if the obtained solution is global.

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