

## BOOK REVIEW

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**Herbert ZEITLER – Dušan PAGON:** *Kreisgeometrie – gestern und heute. Von der Anschauung zur Abstraktion (Circle Geometry – Yesterday and Today. From Visuality to Abstraction)*, 2007 by WBG (Wissenschaftliche Buchgesellschaft), Darmstadt, pp. 195 + XI, Figures and pictures, Additional literature, Register

This book is a masterpiece of the authors, as a scientific and didactic credo, illustrated by the classical inversive geometry. The authors discuss, however, questions of the modern axiomatics as well, from different points of view. The book consists of four main chapters I–IV, divided into sections as follows.

### **I. Elementary geometry**

#### **1. An excursion into geometrical optics**

#### **2. The circle reflection**

The classical inversion and its properties

#### **8. Ratio and cross ratio**

#### **9. An excursion into pencil theory**

#### **10. Sphere reflection**

Applications

#### **12. Circle and sphere reflection as a trick for proofs**

Miquel and pencil theorem (Büschelsatz) – Steiner circle chains – Soddy spheres – circle constructions – Apollonius problem – Steiner sphere chains

## II. Geometry by analytic-algebraic tools

### 1. Analytic geometry in the sense of Descartes

### 2. Analytic geometry in the sense of Gauss

On algebraic structures; group, field and its extension – definitions of complex numbers  $\mathbf{C}$  – the Gauss plane – the Riemann number sphere – points, lines and circles over  $\mathbf{C}$  – reflections and orthogonality – product of reflections – homographies and antihomographies.

### 3. Geometry over field pairs $(K, L)$

Quadratic field extension – finite  $(K, L)$  geometries,  $(K, L)$  – cycle and reflection in it – Miquel’s and other classical theorems as analogues

## III. The affine geometry

### 1. What is axiomatics?

The axiom system  $\Sigma$ , theory  $\text{Th}(\Sigma)$ , model  $\text{Mod}(\Sigma)$  – the theorems of Gödel–Desargues’ and Pappus’ theorems

### 3. The axiom system of the affine plane

### 5. Mappings of the affine plane

### 6. Mappings and configuration theorems

Constructions of coordinate field  $K$ .

## IV. Möbius geometry

### 1. The axiom system of Möbius plane

Various models – views onto high peaks – analogues of former theorems.

Each chapter starts with repetitions of school geometry or hints to former topics, with nice motivations or applications, inspiring the interested readers, mainly the students, teachers to whom this book is highly recommended. The readers “climb high mountains with attractive peaks”. The history of each topic is coloured with the biographies of initiators. Namely, Jakob Steiner (1796–1863), René Descartes (1596–1650), Carl Friedrich Gauss (1777–1855), Karl Christian von Staudt (1798–1867), Benjamino Segre (1903–1977), Ferenc Kárteszi (1907–1989), with warm words about his book *Introduction to Finite Geometries*, as didactic model for the authors), Kurt Gödel (1906–1978), David Hilbert (1862–1943), August Ferdinand Möbius (1790–1868) and others are mentioned with human characteristics.

The popular comparisons of axiomatics and chess play rules are also very convincing. Every chapter ends with problems to solve, some open problems are exposed as well.

The roles and connections of configuration theorems are clearly

discussed. Here certain criticism may have right with respect of didactics. The authors implicitly follow Emil Artin's famous book *Geometric Algebra* in defining affine collineations much before the small and big theorems of Desargues, although the existences of such mappings, and so the existence of coordinate field  $K$ , are just guaranteed by these theorems. The Hungarian readers may miss János Bolyai from the founders of axiomatic geometry. (I noticed also a missing segment 13 from Fig. IV, 3.)

However, the clear overview and comparison in Fig. IV, 14 worth mentioning as well. The newer structure results from books of Peter Dembowski and of Walter Benz have been cited for expert readers.

The summaries of chapters are very useful. The epilogue, how the abstraction appears in human culture, e.g. in the artistic painting, makes this book also very attractive and lovely. The reviewer warmly recommends the English translation of this excellent book.