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DIFFERENTIALLY TRIVIAL NOETHERIAN SEMIPERFECT RINGS

O.D. **Artemovych**

Department of Algebra and Topology, Faculty of Mechanics and Mathematics, Ivan Franko National University of Lviv, University St 1, 79000 Lviv, Ukraine

Dedicated to Professor R. Wiegandt on his 70^th^ birthday

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Abstract: We describe the left Noetherian semiperfect rings whose all proper ideals possess the only trivial derivations and the Noetherian semiperfect rings whose all proper quotient rings possess the only trivial derivations.

1. Let R be an associative ring. A map $d : R \longrightarrow R$ is called a derivation of R if

$$d(a+b) = d(a) + d(b) \text{ and } d(ab) = d(a)b + ad(b)$$

for all $a, b \in R$. Rings having no non-zero derivations will be called here differentially trivial (see [3]).

Note that the notion of differentially trivial ring is in some sense dual to the notion of differentiably simple ring, i.e. ring without proper non-trivial differential ideals, introduced by E.C. Posner (see [19] and [5]).

Earlier, different authors made similar investigations devoted to the rigid rings, i.e. the rings with only identity and zero ring endomor-

E-mail address: artemovych@franko.lviv.ua

phisms. C.J. Maxson [14] and K.R. McLean [15] have described the rigid Artinian rings. M.A. Suppa [17–18], M.D. Friger [11-12] and the author [1–2] have studied I-rigid and q-rigid rings, i.e. rings all proper ideals (respectively all proper quotient rings) of which are rigid.

In this paper we study the differentially i-trivial (respectively differentially q-trivial) rings, i.e. the rings R in which every proper two-sided ideal (respectively every quotient ring R/I, where I is a non-zero ideal of R) is differentially trivial. The main results are the characterizations of differentially i-trivial left Noetherian semiperfect rings and differentially q-trivial Noetherian semiperfect rings.

For convenience of the reader we recall some notation and terminology.

$J(R)$ will always denote the Jacobson radical of a ring R, $\mathrm{Nil}(R)$ the set of all nilpotent elements of R, $\mathrm{char}(R)$ the characteristic of R, $Z(R)$ the centre of R, $H(R)$ the heart of R and $Q(A)$ the field of quotients of a commutative domain A. Throughout the paper p is a prime. In the sequal we will use the following notation:

$\mathbb{Z}_{p^t}$ is the ring of integers modulo a prime power p^t;

$\mathrm{Ann}(I) = \{r \in R \mid rI = 0\}$ is the left annihilator of an ideal I in R.

Let us recall that a ring R is called semiperfect if the quotient $R/\mathcal{J}(R)$ is left Artinian and all idempotents of $R/\mathcal{J}(R)$ can be lifted modulo $\mathcal{J}(R)$ to idempotents of R. A ring R with an identity element is said to be a local ring if $R/\mathcal{J}(R)$ is a skew field.

We also need the following results.

Proposition 1.1. [3]. *Let A be a commutative domain with an identity element.*

(1) *If $\mathrm{char}(A) = p$, then A is differentially trivial if and only if $A = A^p$, where $A^p = \{x^p \mid x \in A\}$.*
(2) *If $\mathrm{char}(A) = 0$, then A is differentially trivial if and only if $Q(A)$ is an algebraic extension of P, where P is the prime subfield of $Q(A)$.*

As defined in I.S. Cohen [7], v-ring V is a commutative unramified complete (in the $\mathcal{J}(V)$-adic topology) regular local rank one domain of characteristic zero with the residue field of characteristic p.

Proposition 1.2. [3]. *Let R be a differentially trivial complete (in the $\mathcal{J}(R)$-adic topology) local left Noetherian ring.*

(1) *If $\mathrm{char}(R) = \mathrm{char}(R/\mathcal{J}(R))$, then R is a field.*

(2) *If* $\operatorname{char}(R) = p^s$ $(s \geq 2)$, *then* $R = \mathbb{Z}_{p^s}$.
(3) *If* $\operatorname{char}(R) = 0$ *and* $\operatorname{char}(R/\mathcal{J}(R)) = p$, *then* R *is a v-ring such that*
$R/p^m R \cong \mathbb{Z}_{p^m}$ $(m \in \mathbb{Z})$.

We will also use some other terminology from [4], [8] and [20].

2. In this part we characterize the differentially i-trivial left Noetherian semiperfect rings. It is obvious that every differentially trivial ring is commutative.

Lemma 2.1. *A nilpotent ring* N *is differentially trivial if and only if* $N = \{0\}$.

Proof. Suppose that N is a non-zero differentially trivial nilpotent ring. Let t be the nilpotency index of N. Then the rule

$$dr = \begin{cases} br, & \text{where } b \text{ is a fixed non-zero} \\ & \text{element of } N^{t-2} \setminus \operatorname{Ann}(N), \text{ if } t > 2, \\ r, & \text{if } t = 2, \end{cases}$$

determines a non-zero derivation $d : N \to N$ of N, a contradiction. The lemma is proved. $\Diamond$

Lemma 2.2. *If* R *is a non-simple differentially* i*-trivial ring with the identity element* 1, *then* $\operatorname{Nil}(R) = \{0\}$.

Proof. Let i be a nilpotent element of R and M a non-zero proper ideal of R. Since M is differentially trivial and the rule $\partial_i(a) = ia - ai$ $(a \in M)$ determines a derivation ∂_i of M, $ai = ia$ for all elements a in M. Therefore iM is a nilpotent ideal and, by Lemma 2.1, $iM = \{0\}$. In the same manner we can prove that $i \cdot \operatorname{Ann}(M) = \{0\}$. Since the ideal $M \cap \operatorname{Ann}(M)$ is nilpotent, in view of Lemma 2.1, we obtain that $M \cap \operatorname{Ann}(M) = \{0\}$. If M is a maximal ideal of R, then $R = M \oplus \operatorname{Ann}(M)$ and, consequently, $i \cdot R = \{0\}$, which implies $i = 0$ because $1 \in R$. The lemma is proved. $\Diamond$

Theorem 2.3. *Let* R *be a non-simple ring with an identity element. If* R *is a differentially* i*-trivial ring, then it is differentially trivial.*

Proof. Suppose this is not true and d is a non-zero derivation of R. Since $I \cap \operatorname{Ann}(I) = \{0\}$ for any ideal I of R, we see that $Ia = \{0\}$ for every $a \in \operatorname{Ann}(I)$ and therefore $\operatorname{Ann}(I)$ is contained in the two-sided annihilator of I. If $Ib = \{0\}$ for some $b \in R$, then $(bI)^2 = b(Ib)I = \{0\}$ and so $bI = \{0\}$. This implies that $\operatorname{Ann}(I)$ is the two-sided annihilator of I in R.

1) Assume that there exists a proper ideal I of R such that $d(I) \neq$ $\neq \{0\}$. Since R does not contain a non-zero nilpotent element and I is commutative, we obtain that $Id(I) = d(I)I = \{0\}$. If $I \oplus \mathrm{Ann}(I) = R$, then $1 = e + f$ for some idempotents $e \in I$, $f \in \mathrm{Ann}(I)$ and $ei = i$ for any $i \in I$. Then $d(i) = d(e)i + ed(i) = 0$, a contradiction. Hence $I \oplus \mathrm{Ann}(I) \neq R$. But then jd is a derivation of $I \oplus \mathrm{Ann}(I)$ for every $j \in I \oplus \mathrm{Ann}(I)$ and so $jd(s) = 0$ for all $s \in I \oplus \mathrm{Ann}(I)$, which gives that $I \oplus \mathrm{Ann}(I)$ is a d-invariant non-zero proper ideal of R, a contradiction.

2) Now, assume that $d(I) = \{0\}$ for any proper ideal I of R. Then, in view of our assumption, it is clear that R contains only unique maximal ideal M and there is an element z of R such that $d(z) \neq 0$. Then $0 = d(mz) = md(z)$ for every m in M and so $d(z) \in \mathrm{Ann}(M)$. Hence $R = M \oplus \mathrm{Ann}(M)$, a contradiction.

Thus, from the previous remark it follows that R has the only trivial derivations. The result is proved. ◊

Proposition 2.4. *Let A be a commutative domain with an identity element. If A is not a field, then the following statements are equivalent:*

(1) *A is a differentially i-trivial ring;*
(2) *A is a differentially trivial ring;*
(3) *A is a commutative ring such that $A = A^p$ if* $\mathrm{char}(A) = p$ *and $Q(A)$ is an algebraic extension of its prime subfield P if* $\mathrm{char}(A) = 0$.

Proof. (1) $\Rightarrow$ (2) follows from Th. 2.3. (2) $\Rightarrow$ (3) follows from Prop. 1.1.

(3) $\Rightarrow$ (1). If $\mathrm{char}(A) = p$, then the Proposition is easily shown. We suppose, therefore, that $\mathrm{char}(A) = 0$. Let I be a non-trivial proper ideal of A and I has a non-zero derivation d. Then the map $D : Q(A) \rightarrow$ $\rightarrow Q(A)$ given by the rule

$$D(r) = i^{-1}(d(j) - d(i)r) \ (r \in A, i, j \in I),$$

where $ir = j$, determines a non-zero derivation D of $Q(A)$. Therefore the extension $P \subseteq Q(A)$, where P is the prime subfield of $Q(A)$, is not algebraic [6, Chapter V, §9, Cor. 2], a contradiction, which completes the proof. ◊

Lemma 2.5. *Let R be a differentially trivial domain of characteristic zero with an identity element. If R contains a subfield, then $\mathcal{J}(R)$ is trivial.*

Proof. Since R contains a subfield, its prime subfield P is isomorphic to the rational number field $\mathbb{Q}$. Then, by Prop. 1.1, for every element

j of $\mathcal{J}(R)$ there exists a non-zero polynomial

$$f(x) = \sum_{i=0}^{n} a_i x^{n-i} \in P[x]$$

such that $f(j) = 0$. Hence

$$a_n = -\sum_{i=0}^{n-1} a_i j^{n-i} \in P \cap \mathcal{J}(R),$$

and this yields that

$$j^k \cdot (\sum_{i=0}^{n-k} a_i j^{n-i-k}) = 0,$$

where $a_{n-k} \neq 0$ and $a_i = 0$ for $i > n-k$. Then $j^k = 0$ because $\sum_{i=0}^{n-k} a_i j^{n-i-k}$ is an invertible element in R, and we have a contradiction. The lemma is proved. $\Diamond$

Corollary 2.6. *If a local Noetherian ring R is differentially i-trivial, then it is a domain.*

Proof. Let us assume that the result is false and R is not a domain. Then, by Th. 2.3, R is a commutative ring and, by Cor. 1 of [6, Chapter IV, §1, no. 1], there exists a prime ideal $P = \mathrm{Ann}(x)$ for some non-zero element x of R. If d is a non-zero derivation of R/P and $d(r+P) = h+P$, with $r, h \in R$, then the rule $D(r) = xh$ determines a non-zero derivation D of R. From this, in view of Th. 2.3, it follows that R/P is a differentially trivial ring.

If $\mathrm{char}(R/P) = 0$, then, in view of Lemma 4.5.1 from [10], R/P contains a subfield and then, by Lemma 2.5, $P = \mathcal{J}(R)$. If $\mathrm{char}(R/P) = p$ for some prime p, then, by Prop. 1.1, $(R/P)^p = R/P$. Hence $(\mathcal{J}(R/P))^p = \mathcal{J}(R/P)$ and, in view of Prop. 8.6 of [4], $P = \mathcal{J}(R)$.

Thus, in both cases $x \in \mathcal{J}(R) \cap \mathrm{Ann}(\mathcal{J}(R))$ and so $x^2 = 0$, which leads to a contradiction with Lemma 2.2. The result follows. $\Diamond$

Corollary 2.7. *Let R be a differentially i-trivial local Noetherian ring.*

1) *If* $\mathrm{char}(R) = \mathrm{char}(R/\mathcal{J}(R))$*, then R is a skew field.*

2) *If* $\mathrm{char}(R) \neq \mathrm{char}(R/\mathcal{J}(R))$*, then R is a differentially trivial domain of characteristic* 0 *and* $\mathrm{char}(R/\mathcal{J}(R)) = p$ *for some prime p.*

Proof. By Cor. 2.6, R is a domain. Suppose that $\mathcal{J}(R) \neq \{0\}$. Then, by Th. 2.3, R is differentially trivial and, consequently, commutative.

1) Let $\mathrm{char}(R) = \mathrm{char}(R/\mathcal{J}(R))$. If $\mathrm{char}(R) = 0$, then, by Lemma 4.5.1 of [10], R contains a subfield and so, by Lemma 2.5, $\mathcal{J}(R) = \{0\}$,

a contradiction. Assume that $\mathrm{char}(R) = p$ for some prime p. Then, in view of Cor. 2.6 and Prop. 1.1, $\mathcal{J}(R)^p = \mathcal{J}(R)$ and therefore, by Krull Theorem, $\mathcal{J}(R) = \{0\}$, a contradiction. Thus, if $\mathrm{char}(R) = \mathrm{char}(R/\mathcal{J}(R))$, then $\mathcal{J}(R) = \{0\}$.

2) Now, let $\mathrm{char}(R) \neq \mathrm{char}(R/\mathcal{J}(R))$. Then, in view of Lemma 2.2, $\mathrm{char}(R) = 0$ and $\mathrm{char}(R/\mathcal{J}(R)) = p$ for some prime p. Finally, by Th. 2.3, R is a differentially trivial ring. The corollary is proved. $\Diamond$

Theorem 2.8. *Let R be a left Noetherian semiperfect ring. Then R is a differentially i-trivial ring if and only if it is of one of the following types:*

(1) *R is a skew field;*
(2) *R is a differentially trivial local Noetherian domain of characteristic 0 and $\mathrm{char}(R/\mathcal{J}(R)) = p$ for some prime p;*
(3) *$R = R_1 \oplus \ldots \oplus R_m$ $(m \geq 2)$ is a finite ring direct sum of $R_1, \ldots, R_m$ and each R_i is either a differentially trivial field, or a ring of type (2).*

Proof. $(\Leftarrow)$ is obvious.

$(\Rightarrow)$. If R is a local ring, then, by Cor. 2.6, it is a ring of type (1) or (2). Therefore we assume that R is not local and so, by Th. 2.3, it is a commutative ring. Since every commutative semiperfect ring is a finite ring direct sum of local rings (see e.g. [9, Ex. 22.27]), the claim follows from Th. 2.3, Prop. 2.4 and Cor. 2.7. $\Diamond$

3. In this section our object is to study the differentially q-trivial Noetherian semiperfect rings.

It is clear that every simple ring is differentially q-trivial. Recall that the heart $H(R)$ of a ring R (if it exists) is the smallest non-zero ideal of R.

Lemma 3.1. *Let R be a non-simple ring. If R is differentially q-trivial, then R is either commutative or contains the heart $H(R)$.*

Proof. In fact, since R/I is differentially trivial by definition, $[R,R] \subseteq I$ for every non-zero ideal I of R, where $[R,R] = \{xy - yx \mid x, y \in R\}$, and, consequently, either R is a commutative ring or $\{0\} \neq [R,R] \leq H(R) = \cap\{I \mid I \text{ is an ideal of } R\}$. The lemma is proved. $\Diamond$

Let R be an associative ring with two operations " $+$ " and " $\cdot$ ". Recall that a ring R is called radical (or equivalently, quasiregular) if the set of all elements from R forms a group with the identity element $0 \in R$ under the circle operation " $\circ$ ", defined by the rule $a \circ b = a + b + a \cdot b$ for all elements a and b in R. As it follows from [16], a homomorphic image of the ring $\mathbb{Q}[[X,Y]]$ of formal power series in two noncommuting indeterminates X and Y over the rational number field $\mathbb{Q}$ contains a minimal ideal $\mathfrak{A}$, which is a simple radical ring. Moreover, it is easy to see that the additive group $\mathfrak{A}^+$ is divisible.

Lemma 3.2. *Let R be a local Noetherian ring. Then R is differentially q-trivial if and only if it is of one of the following types:*

(1) $R \cong \mathbb{Z}_{p^n} \quad (n \in \mathbb{N})$;
(2) $R \cong B[x]/(x^2)$, *where $B[x]$ is a commutative ring in an indeterminate x over a differentially trivial field B;*
(3) *R is a non-commutative Artinian ring with $\mathcal{J}(R) = H(R)$ and the differentially trivial residue field $R/\mathcal{J}(R)$;*
(4) *R is a v-ring such that $R/p^mR \cong \mathbb{Z}_{p^m} \quad (m \in \mathbb{N})$;*
(5) *R is a skew field;*
(6) *$R = V + H(R)$ is a group direct sum of a differentially trivial v-ring V and the heart $H(R)$, which is a simple Noetherian radical ring with the divisible additive group $H(R)^+$, $\mathcal{J}(R) = pV + H(R)$ and $V \leq Z(R)$.*

Proof. Let R be a differentially q-trivial local Noetherian ring, W the subring of R generated by its identity element. Then $W \leq Z(R)$ and so $W \cap H(R) = \{0\}$.

1) Assume that the Jacobson radical $\mathcal{J}(R)$ is nilpotent of the nilpotency index $n \geq 2$ and, consequently, R is complete (in the $\mathcal{J}(R)$-adic topology) (see e.g. [7]).

If $n \geq 3$, then $B = R/\mathcal{J}(R)^{n-1}$ is not a field. In view of Prop. 1.2, $\mathrm{char}(B) \neq 0$ and, consequently, $B \cong \mathbb{Z}_{p^{n-1}}$. Thus $R = W + \mathcal{J}(R)^{n-1}$. If $W = R$, then $R \cong \mathbb{Z}_{p^n}$. Therefore we assume that $W \neq R$. Then, as a consequence of Prop. 1.2(2), the quotient ring $R/(W \cap \mathcal{J}(R)^{n-1})$ has a non-zero derivation, a contradiction.

Now, let $n = 2$.

a) Assume that R is a commutative ring. Then, by results of I.S. Cohen [7, Ths. 9 and 11], $R = D + \mathcal{J}(R)$, where D is some subring of R (in [7] and [20] D is called a coefficient ring of R).

a_1) If $\mathrm{char}(R) = \mathrm{char}(R/\mathcal{J}(R))$, then D is a field (see [7, Th. 9] or [20, Chapter VIII, §12, Th. 27]) and the quotient ring $R/iR = \overline{D} + \mathcal{J}(\overline{R})$ is differentially trivial for every non-zero $i \in \mathcal{J}(R)$. By Prop. 1.2(1), R/iR is a field and hence $\mathcal{J}(R) = iR = iD$. Thus we have

$$R = D + iD \cong D[x]/(x^2),$$

where D is a differentially trivial field, i.e. R is a ring of type (2).

a_2) If $\operatorname{char}(R) \neq \operatorname{char}(R/\mathcal{J}(R))$, then R is a ring of prime power characteristic p^2. Then $D \cong V/p^2V$ for some v-ring V [7, Th. 11]. Since $D/pD \cong V/pV$ is differentially trivial, $D \cong \mathbb{Z}_{p^2}$ (see Prop. 1.2). Consequently, $D \cap \mathcal{J}(R) \neq \{0\}$, the quotient ring

$$\overline{R} = R/(D \cap \mathcal{J}(R)) = \overline{D} + \mathcal{J}(\overline{R})$$

is differentially trivial by our hypothesis, and therefore, by Prop. 1.2(1), $\mathcal{J}(\overline{R}) = \{\overline{0}\}$. This means that $R = D$ is a ring of type (1).

b) Now, assume that the ring R is non-commutative. By Lemma 3.1, R has the heart $H(R)$, and then, by our hypothesis, the quotient ring $R/H(R)$ is differentially trivial. Since R is a non-commutative ring and $\mathcal{J}(R)$ is a nilpotent ideal, we conclude that $R/H(R)$ is a field or $R/H(R)$ is isomorphic to some $\mathbb{Z}_{p^t}$ $(t \geq 2)$. If $R/H(R) \cong \mathbb{Z}_{p^t}$, then $pH(R) = \{0\}$ and pW is a proper non-zero ideal of R, which leads to a contradiction. Hence $R/H(R)$ is a field and so $H(R) = \mathcal{J}(R)$. From $rH(R) = H(R)r = H(R)$ for all $r \in R \setminus H(R)$, we obtain, by Robson Theorem [9, Th. 20.35], that R is an Artinian ring of type (3).

2) Now, let $\mathcal{J}(R)$ be a non-nilpotent ideal.

c) If R is a commutative ring, then, by Krull Theorem,

$$\bigcap_{n=1}^{\infty} \mathcal{J}(R)^n = \{0\}.$$

This implies that $R/\mathcal{J}(R)^n$ is commutative and complete (in the $\mathcal{J}(R)$-adic topology) (see e.g. [7]) and, by Prop. 1.2, $R/\mathcal{J}(R)^n \cong \mathbb{Z}_{p^n}$. Therefore we conclude that R is a differentially trivial v-ring, i.e. it is a ring of type (4).

d) If R is a non-commutative ring, then, by Lemma 3.1, there is the heart $H(R)$ and so the quotient ring $\overline{R} = R/H(R)$ is differentially trivial. If $\operatorname{char}(R) \neq 0$, then, by Th. 4.5.3 of [10], $\operatorname{char}(R) = p^m$ for some prime p and integer m. Therefore $W \cong \mathbb{Z}_{p^m}$ and $pH(R) = \{0\} = pWH(R)$, which leads to a contradiction. Hence $\operatorname{char}(R) = 0$ and, as in the part *c*), $\overline{R} \cong W$ is a differentially trivial v-ring. If $H(R)^2 = \{0\}$, then $H(R)$ is the Wedderburn radical of R and $jH(R) = H(R)j = H(R)$ for all $j \in R \setminus H(R)$. Robson Theorem [9, Th. 20.35] now easily yields that $H(R) = \{0\}$, a contradiction. Hence $H(R) = H(R)^2$ is a simple Noetherian radical ring and therefore R is a ring of type (6).

The converse is obvious and the lemma is proved. ◊

Remark 3.3. Let $F = GF(p^n)$ be a finite field $(n \geq 2)$, σ the Frobenius map of F and $F[x;\sigma]$ a skew polynomial ring, i.e. $xa = a^{\sigma}x$ for every

$a \in F$. Then the quotient ring $F[x;\sigma]/(x^2)$ is differentially q-trivial, but it is not differentially trivial (see Lemma 3.2(3)).

Theorem 3.4. *Let R be a Noetherian semiperfect ring. Then R is a differentially q-trivial ring if and only if it is of one of the following types:*

(1) *R is a skew field;*
(2) $R \cong \mathbb{Z}_{p^n}$ $(n \in \mathbb{N})$;
(3) *R is a v-ring such that $R/p^m R \cong \mathbb{Z}_{p^m}$ $(m \in \mathbb{N})$;*
(4) *$R \cong B[x]/(x^2)$, where $B[x]$ is a commutative ring in an indeterminate x over a differentially trivial field B;*
(5) *R is a non-commutative Artinian local ring with the $\mathcal{J}(R) = H(R)$ and the differentially trivial residue field $R/\mathcal{J}(R)$;*
(6) *$R = V + H(R)$ is a group direct sum of a differentially trivial v-ring V and the heart $H(R)$, which is a simple Noetherian radical ring with the divisible additive group $H(R)^+$, $\mathcal{J}(R) = pV + H(R)$ and $V \leq Z(R)$;*
(7) *$R = R_1 \oplus \ldots \oplus R_m$ $(m \geq 2)$ is a ring direct sum of $R_1, \ldots, R_m$ and each R_i is either a differentially trivial field, or differentially trivial v-ring, or isomorphic to some $\mathbb{Z}_{p^t}$;*
(8) *R is a non-commutative ring, which contains the heart $H(R)$ and $R/H(R)$ is a ring of type (7).*

Proof. $(\Leftarrow)$ is obvious.

$(\Rightarrow)$: If R is a local ring, then, by Lemma 3.2, it has one of types (1)$-$(6). Therefore we assume that R is not local. If R is commutative, then, as a consequence, it is a ring direct sum of differentially trivial local Noetherian rings and, in view of Lemma 3.2, R is a ring of type (7). Assume that R is a non-commutative ring and I_0 its smallest ideal. Then $\{0\} \neq H(R) = \mathcal{J}(R)$ and $R/H(R)$ is a differentially trivial ring of type (7). The proof is finished. $\Diamond$

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