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ON NEAR-RINGS WITH ACC ON ANNIHILATORS

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Abstract: Our attempt is on what may be termed as parts with minimum condition in case of a near-ring N with ascending chain condition (acc) on left annihilators and having no infinite independent family of left N-subgroups of it.

We prove the inheritance of descending chain condition (dcc) of a near-ring modulo the left annihilator of a minimal countable left ideal with dcc on its left N-subgroups. An invariant subnear-ring, minimal as a left N-subgroup and is with dcc on its right N-subsets is, in some sense, a left near-ring group over a near-ring which is an extension of an epimorphic image of N. Also we prove some results on $s - A(N)$ the sum of all left ideals (of N) with dcc on left N-subgroups. In some cases $s - A(N)$ is not contained in any minimal strongly prime ideal of N. In case $s - A(N)$ is a minimal countable invariant subnear-ring with dcc on left N-subgroups of the minimal strongly prime ideals, $s - A(N)$ is a direct summand of N.

1. Introduction

A. Oswald in [7] has obtained the structure of near-rings in which each near-ring subgroup is principal. The results, though he describes as an analogous result to what A. W. Goldie obtained on the structure of semiprime rings [5], it has some independent beauty of its own. M. N. Baruah in his thesis [1] has given, according to Laxton, an elegant generalization dealing with the theory of right quotients of a right near-ring. Seth and Tiwari [10] have established the existence of the near-ring of left quotients of a right near-ring.

Our object in this paper is to prove some results on substructures (with various types of minimum conditions) of a near-ring with chain conditions as discussed by Oswald [8], Chowdhury [3] as well as to deal with some interesting properties of such a near-ring with some radical character.

Any near-ring with ascending chain condition (acc) on left near-ring subgroups obviously satisfies the conditions what Oswald has chosen (viz., no infinite direct sum of left ideals and satisfies the ascending chain condition (acc) on left annihilators). But rings like $Z[X_i | i = 1, 2, \ldots; X_i X_j = X_j X_i]$ are such that these satisfy the acc on left annihilators having no infinite direct sum of left ideals but we have a strictly ascending infinite chain of ideals such as

$$(X_1) \subset (X_1, X_2) \subset \ldots.$$

Thus near-rings with the conditions described in [8] need not satisfy the acc on its subalgebraic structures. Yet we may have parts of such near-rings satisfying maximum or minimum conditions on its substructures. Towards the later part of the paper we obtain results on near-rings with this radical character. In particular we deal with $s - A(N)$ (strictly Artinian radical) the sum of all left ideals of a near-ring N satisfying the descending chain condition (dcc) on left N-subgroups.

All basic concepts used in the paper are available in Pilz [9] and throughout the paper N will mean a (zero symmetric) distributively generated right near-ring (dgnr) with 1 and with the following two conditions

(1) N has no infinite independent family of left N-subgroups of N;
(2) N satisfies the acc on left annihilators of subsets of N.

2. Definitions and notations

An element $x \in N$ is *regular* if for some $y \in N$, $xyx = x$ and N is *regular* if each element of N is regular. An *invariant subset* A of N is such that $A \subseteq N$ and NA, $AN \subseteq A$. N is *strongly semiprime* if N has no nonzero nilpotent invariant subset. It is easy to see that a regular near-ring is always strongly semiprime. A partial converse of it is seen in [8, Cor. 4]. E will denote a left N-group of N^E, a left N-subgroup A of N will mean an N-subgroup of N^N and a left ideal of N will mean an ideal of N^N. If A and B are two left N-subgroups of N, $A \subseteq B$, then A is an *essential left N-subgroup* of B, when any nonzero left N-subgroup $C(\subseteq B)$ has a nonzero intersection with A. A left N-subgroup A of N is a *weakly essential left N-subgroup* of N if it has nonzero intersection with any nonzero left ideal of it. It is clear that an essential left N-subgroup of N is always a weakly essential left N-subgroup of it. We note that in the symmetric group S_3, [9, (37), p. 342], $\{0, a\}$, $\{0, b\}$, $\{0, c\}$ and $\{0, x, y\}$ are proper nonzero left N-subgroups where $\{0, x, y\}$ is a left ideal. So this is not an essential left N-subgroup though it is weakly essential. Thus a weakly essential left N-subgroup of N need not be an essential left N-subgroup of it. A near-ring N is *left non singular* if $Z_l(N)$ $(= \{x \in N | Ix = 0$, for some essential left N-subgroup I of $N\}) = 0$. A *right annihilator* of $S(\subseteq N)$ is defined as $\mathrm{r}(S) = \{n \in N | Sn = 0\}$. Similarly a *left annihilator* $\mathrm{l}(S)$ is defined.

A left N-subgroup A of N is a *closed N-subgroup* of N is A is an essential left N-subgroup of a left N-subgroup $B(\subseteq N)$ implies $A = B$. The collection of all maximal left annihilators of the type $P = \mathrm{l}(A)$ where A is a left N-subgroup of N will be denoted by $\mathcal{P}$. The near-ring N satisfies (left) *Ore condition* with respect to a subset S of it if for given $(r, a) \in S \times N$ there exists a common left multiple $r'a = a'r$ such that $(r', a') \in S \times N$. Now we note the following

Note 1. A strongly semiprime near-ring N can not have any nonzero nilpotent left (right) N-subset and therefore it has no nonzero nil left (right) N-subset.

Note 2. For any essential left N-subgroup A of N, $Ax^{-1} = \{n \in$ $\in N | nx \in A\}$ (for nonzero $x \in N$) is an essential left N-subgroup of N.

Note 3. If N is strongly semiprime, such that an essential left ideal is an essential left N-subgroup of N, then N satisfies the dcc on left annihilators. Also $\mathrm{r}(P_i) \subseteq P_j$ $(i \neq j)$, $T_i \cap T_j = 0$ for $P_i = \mathrm{l}(T_i)$, $P_j = \mathrm{l}(T_j)$, $P_iP_j \in \mathcal{P}$.

In near-rings 2.2,([4, p. 390], [9, (20)]), a weakly essential left N-subgroup of N is an essential left N-subgroup of it. In what follows, we confine our discussion to those N where weakly essential left N-subgroups are essential — an extension of what is described in Note 3, as in [8, Th. 7].

3. Preliminaries

Lemma 3.1. *If N is strongly semiprime, $P \in \mathcal{P}$ then the near-ring N/P $(= \overline{N})$ also satisfies the acc on left annihilators and has no infinite independent family of left N-subgroups.*

Proof. The proof follows from the fact that a left annihilator of a subset of $\overline{N}$ is of the type $\overline{J} = \mathrm{l}(\overline{T})$ where $J = \mathrm{l}(\mathrm{Tr}(P))$ and an independent family $\{\overline{J}_1\}$ of left N-subgroups gives an independent family $\{I_i = A \cap$ $\cap J_i\}$ of left N-subgroups of N $(P = \mathrm{l}(A))$. $\Diamond$

If M is any left N-subgroup of N and $x \in N$ is such that $\mathrm{l}(x) =$ $= 0$, then it is easily seen that $(\sum_{n \neq t} Mx^n) \cap Mx^t = 0$ $(0 \leq t < s$, $n = 0, 1, \ldots, s)$. Thus,

Lemma 3.2. *If for $x \in N$, $\mathrm{l}(x) = 0$, then Nx is an essential left N-subgroup of N.*

In Klein's four group, [9, p. 340], the near-ring (11) is left non singular whereas the near-ring (14) is not. Moreover in both cases, left annihilators $(\neq N)$ are distributively generated (viz. $\{0, b\}$). In (11), the left N-subgroup $\{0, a\}$ is not weakly essential containing zero-divisors only and in (14), left N-subgroups $\{0, a\}$ and $\{0, e\}$ are also of same character with $\{0, b\}$ as an exception in both cases. In the following lemma we see how strongly semiprime characters (and hence left non singularity of N) together with distributively generated left annihilators play key role for the existence of a non-zerodivisor in a weakly essential left N-subgroup.

Lemma 3.3. *N is strongly semiprime with distributively generated left annihilators. Then a weakly essential left N-subgroup of N contains a non-zerodivisor.*

Proof. If I is any weakly essential left N-subgroup of N, then it is not nil. So, it is possible to choose an element $c = a_1 + \cdots + a_n (\in I)$ with each a_i non nilpotent such that $\mathrm{l}(c) = 0$ and left nonsingularity of N gives that $\mathrm{r}(c) = 0$. ◊

If N has distributive non-zerodivisors, then for $a \in S$ (semigroup of distributive non-zerodivisors), $b \in N$, Na is an essential left N-subgroup of N and so $(Na)b^{-1} = \{n \in N | nb \in Na\}$ is an essential left N-subgroup of N and thus it contains a non-zerodivisor (say) a_1. So, $a_1 b = b_1 a$ for some $b_1 \in N$. And this left common multiple properly with respect to S leads towards the coincidence of complete near-ring of left quotient of N with classical near-ring Q of left quotients with respect to S [10]. We now give

Lemma 3.4. *Q satisfies the dcc on its left Q-subgroups.*

Proof. For any two left Q-subgroups A, B of Q with $B \subseteq A$ and $B \cap N$ is essential left N-subgroup of $A \cap N$, we get $A = B$ (Lemma 3.3). Thus a strictly descending chain of left Q-subgroups of Q gives rise to an independent family of left N-subgroups of N and thus such an infinite descending chain contradicts the character of N. ◊

Lemma 3.5. *For a strongly semiprime near-ring N, the collection $\mathcal{P}$ of all P's described above is a finite one and $\bigcap_{P \in \mathcal{P}} P = 0$.* ◊

If for some nonzero left N-subgroup A of a dgnr N, $P = \mathrm{l}(A)$ is a maximal left annihilator then P is an ideal of N and for any invariant subnear-rings B, C of N, $BC \subseteq P$ we have either $B \subseteq P$ or $C \subseteq P$. In this sense P is *strongly prime*. Also P is a *minimal* strongly prime ideal in the sense that for any strongly prime ideal P', $P' \subseteq P$ gives $P' = P$. The collection $\{B_i\}$ of all ideals $B_i (= \mathrm{l}(I_i))$ contained in an ideal $B (= \mathrm{l}(I))$ containing an ideal U, has a maximal element (say) B_2. So $B = B_1 \supseteq B_2 \supseteq U$. Similarly we get ideals $B_3, B_4 \ldots$ such that $B = B_1 \supseteq B_2 \supseteq \ldots$ and each of them contains U. This chain is a finite one. So $B = B_1 \supseteq B_2 \supseteq \ldots \supseteq B_{k-1} = U = B_k$. If $\mathrm{l}(B_j / /B_{j+1}) = P_j / B_{j+1}$, $P_j B_j \subseteq B_{j+1}$, we get $X = P_{k-1} P_{k-2} \ldots P_2 P_1$. Then $XB = (P_{k-1} \ldots P_1)\, B \subseteq (P_{k-1} P_{k-2} \ldots P_2) B_2 \subseteq \ldots \subseteq B_k = U$. Thus we get

Lemma 3.6. *If N is a strongly semiprime near-ring then for an ideal $B (= \mathrm{l}(I))$ containing an ideal U there is an X which is a finite product of ideals such that $XB \subseteq U$.* ◊

In the near-rings N [4, 2.1, 10], $B = \{0, b\}$ is the only proper left ideal of N and it is such that $BN = B$, $NB = B$. Also in [4, 2.2, 13], $A = \{0, a\}$, $B = \{0, b\}$, $C = \{0, c\}$ are left ideals of the corresponding near-ring N and $AB = B$, $BA = 0$, $AC = C$, $CA = A$, $A^2 = A$, $B^2 = 0$, $C^2 = C$. Thus any finite product of left ideals is again a left ideal and in this sense each of the near-rings are $(1, i)$ *closed.*

By Lemma 3.6 we get

Lemma 3.7. *Let N be a strongly semiprime $(1, i)$ closed near-ring and let $B = (\mathrm{l}(I))$ be an ideal of N. If C is a left ideal of N then there is a left ideal Y such that $YB \subseteq BC$. If $P = \mathrm{l}(I)$ is closed then $Z(I) = \{x \in I | cx = 0,$ for some non-zerodivisor $c + P \in N/P\} = 0$.*

Proof. P being a closed left N-subgroup of N, for any essential left N-subgroup A of N $(P \subseteq A)$ A/P is an essential N-subgroup as well as left N/P-subgroup of N/P $(= \overline{N})$. For $x \in Z(I)$ we get $cx = 0$ for some non-zerodivisor $\overline{c} \in N/P$. Because of Lemmas 3.2 and 3.1, $\overline{N}\,\overline{c}$ is an essential left $\overline{N}$-subgroup of $\overline{N}$. Now for $n \in N$, $(\overline{N}\,\overline{c})\overline{n}^{-1} = \{\overline{u} | \overline{u}\,\overline{n} \in \overline{N}\,\overline{c}\}$ is an essential left $\overline{N}$-subgroup of $\overline{N}$. If $\overline{A}$ is a weakly essential left $\overline{N}$-subgroup of $\overline{N}$ so is A of N and therefore A is an essential left N-subgroup of N. As P is closed and $P \subseteq A$, $\overline{A}$ is an essential left $\overline{N}$-subgroup of $\overline{N}$. Thus every weakly essential left $\overline{N}$-subgroup of $\overline{N}$ is essential. Moreover, left annihilators in $\overline{N}$ are also distributively generated. So by Lemma 3.3, $(\overline{N}\,\overline{c})\overline{n}^{-1}$ contains a non-zerodivisor (say) $\overline{d}$. Then $\overline{d}\,\overline{n} = \overline{u}\,\overline{c}$ for some $u \in N$ which gives $(dn - uc)I = 0$. So $(dn - uc)x = 0$ with $x \in Z(I)$. Thus $dnx = 0$ which gives $nx \in Z(I)$. Therefore $Z(I)$ is a left N-subset of N. Moreover, $cx = 0$ gives $\overline{c}\,\overline{x} = \overline{0}$ and this implies $\overline{x} = \overline{0}$ or $xI = 0$. Thus $Z(I)I = 0$ which gives $(Z(I))^2 = 0$ and N being strongly semiprime it follows that $Z(I) = 0$. $\Diamond$

4. Main results

We now prove the main results, the first two being on a strongly semiprime near-ring N with distributively generated left annihilators and others are on $s - A(N)$.

Theorem 4.1. *Let N be a strongly semiprime near-ring with distributive non-zerodivisors. If I is an invariant subnear-ring, minimal as a left N-subgroup of N and is with dcc on its right N-subsets ($A \subseteq I$, $AN \subseteq I$) and $C(P) = C(0)$, $P = \mathrm{l}(I)$, then I is a left near-ring-group over Q, a near-ring with dcc on its left Q-subgroups and an extension of an epimorphic image of N.*

Proof. By Lemma 3.4 and what is described before it, the near-ring Q of left quotients of N/P $(= \overline{N})$ satisfies the dcc on its left Q-subgroups. If $d \in N$ is a non-zerodivisor, and I being with dcc on its right N-

subsets, we get a $t \in \mathbb{Z}^+$ such that, $d^t = d^{t+1}I$ which finally gives $I = dI$ and hence $\overline{I} = \overline{d}^{-1}\overline{I}$, where $\overline{I} = \{i + P | i \in I\}$. Now the map $Q \times \overline{I} \to \overline{I}$ given by $(\overline{d}^{-1}\overline{r}, \overline{x}) \to \overline{d}^{-1}(\overline{r}\,\overline{x})$ makes $\overline{I}$ a left Q-group. Considering the map $Q \times I \to I$ with $(q, i) \to x$ where $q\overline{i} = \overline{x}$ we get x is unique; if $x, y \in I$, $\overline{x} = Y$ then $x - y \in P$ and so $x - y \in I \cap P = 0$ giving thereby $x = y$. Now if $i \in I$, $q_1, q_2 \in Q$ and $q_1\overline{i} = \overline{x}_1$, $q_2\overline{i} = \overline{x}_2$ then $(q_1+q_2)\overline{i} = q_1\overline{i} + q_2\overline{i}$ (as I is a left Q-group). So, $(q_1+q_2)\overline{i} = \overline{x}_1 + + \overline{x}_2 = \overline{x_1 + x_2}$ and this gives $(q_1 + q_2)i = x_1 + x_2 = q_1 i + q_2 i$. Also, $(q_1q_2)i = q_1(q_2 i)$ is easy. Thus I is a left Q-group over the near-ring of left quotient of $\overline{N}$, an epimorphic image of N. $\Diamond$

Theorem 4.2. *If I is a countable left ideal of N satisfying the dcc on its left N-subgroups then the N-group $N/\mathrm{l}(I)$ also satisfies the dcc on its N-subgroups.*

Proof. Because of Note 3 it is possible to find a finite subset $S = = \{y_1, \ldots, y_t\}$ of I such that $\mathrm{l}(I) = \mathrm{l}(S) = \mathrm{l}(y_1) \cap \ldots \cap \mathrm{l}(y_t)$. Now for the homomorphism $f : N \to Ny_1 \oplus \ldots \oplus Ny_t$ where $f(a) = (ay_1, \ldots, ay_t)$, $a \in N$, $\ker f = \mathrm{l}(S) = \mathrm{l}(I)$. So, $f(N) \simeq N/\mathrm{l}(I)$ and we get a monomorphism $N/\mathrm{l}(I) \to Ny_1 \oplus \ldots \oplus Ny_t$. Thus $N/\mathrm{l}(I)$ can be embedded as an N-group in $Ny_1 \oplus \ldots \oplus Ny_t$. Since I satisfies the dcc on its left N-subgroups so is also $Ny_i(\subseteq I)$. Thus $Ny_1 \oplus \ldots \oplus Ny_t$ is also with dcc on its left N-subgroups. Hence $N/\mathrm{l}(I)$ is with dcc on its left N-subgroups. $\Diamond$

In case of $N = \begin{bmatrix} Z & F \\ 0 & Z \end{bmatrix}$, $F = Z/(2Z)$ we **note**: It satisfies that for ideals I and X, N/X with dcc on N-subgroups there is an ideal Y such that N/Y is also with $YI \subseteq IX$ [2, Cor. 4.4, p.57].

The following results apply to a near-ring N as in Lemma 3.7 as above which satisfies the property that if N/X is with dcc on its N-subgroups then so is N/Y. $s - A(N)$ is a left ideal of a dgnr N and since, for two left ideals A, B of N, $\frac{A+B}{B}$ and $\frac{A}{A \cap B}$ are isomorphic, it can be easily seen that $s - A(N)$ satisfies the dcc on its left N-subgroups. $s - A(N) = N$ if N is with dcc on its left N-subgroups and if N has no nonzero left ideals with dcc on its left N-subgroups, then $s - A(N) = 0$.

Now we prove the following

Theorem 4.3. *Let N be a strongly semiprime $(1, i)$ closed near-ring. If P is a minimal strongly prime ideal with dcc on its left N-subgroups and N/P is with dcc on its left N-subgroups then $s - A(N)$ is not contained in P.*

Proof. Let T be the set of all ideals of N such that N/I is with dcc on its (left) N-subgroups and is closed upto finite products. As $P \in$ $\in T$, $T \neq \emptyset$. N has a finite number of minimal strongly prime ideals $P_1, P_2, \ldots P_l$ each of which is an annihilator ideal ($P_i = \mathrm{l}(I_i)$). So $P =$ $= P_i$, for some i. If for all i, $P_i \in T$ then the homomorphism $N \to$ $\to N/P_i \oplus \ldots \oplus N/P_l$ is a monomorphism. As each N/P_i is with dcc on its left N-subgroups, N posseses the same character. So, $s-A(N) = N$. Hence $s - A(N) \not\subseteq P$.

Next, for some j, $P_j \notin T$. If $P_1 \notin T$, $P_2 \in T$, then by Lemma 3.7 there exists $Q_1 \in T$ such that $Q_1 P_1 \subseteq P_1 P_2$. Thus $Q_1 P_1 \ldots P_l \subseteq$ $\subseteq P_1 P_2 \ldots P_l = 0$. Thus by moving $P_j (\notin T)$ to the right we obtain $XY = 0$ where $X = Q_1 \ldots Q_k$, $Q_i \in T$ and $Y = P_1 P_2 \ldots P_r$, $P_i \notin T$. Let $Y \supseteq Y_1 \supseteq Y_2 \ldots$ be a descending chain of left N-subgroups of Y. This gives a descending chain of N-subgroups of N/X where $B_i = \{y_i +$ $+ X | y_i \in Y_i\}$. $XY = 0$ gives $X \cap Y = 0$. So we get $t \in \mathbb{Z}^+$ such that $B_t = B_{t+1}$ which gives the dcc on left N-subgroups of $Y (\subseteq s - A(N))$. But $Y = P_1 \ldots P_r$ where each P_i is strongly (minimal) prime ideal such that $P_i \notin T$. So, $Y \subseteq P$. Thus $s - A(N) \not\subseteq P$. ◊

By Th. 4.2 $N/\mathrm{l}(s - A(N))$ is with dcc on its N-subgroups when $s - A(N)$ is minimal and countable. Now for any minimal strongly prime P, $\mathrm{l}(s - A(N)) \not\subseteq P$ gives $s - A(N) + \mathrm{l}(s - A(N)) \not\subseteq P$. Also $\mathrm{l}(s - A(N)) \subseteq P$ gives $P = \mathrm{l}(s - A(N))$ and therefore N/P is with dcc on its N-subgroups (Th. 4.2). Thus by Th. 4.3, $s - A(N) \not\subseteq P$. So, in any case $s - A(N) + \mathrm{l}(s - A(N)) \not\subseteq P$.

Now, if I is an invariant subnear-ring of N with $I \not\subseteq P$ then for any left N-subgroup L of N with $I \cap L = 0$ we get $LK = 0$ giving thereby $L = 0$, thus I is an weakly essential left N-subgroup of N. Utilizing also Lemma 3.3, we get

Theorem 4.4. *If $s - A(N)$ is minimal, countable invariant sub-near-ring and minimal strongly prime ideals of the strongly semiprime N are with dcc on its left N-subgroups then $s - A(N) + \mathrm{l}(s - A(N))$ contains a non-zerodivisor.* ◊

Theorem 4.5. *If minimal strongly prime ideals of the strongly semi-prime N are with dcc on its left N-subgroups and $s - A(N)$ is a minimal, countable, distributively generated invariant subnear-ring, then $s - A(N)$ is cyclic one generated by an idempotent.*

Proof. By Th. 4.4, $s - A(N) + \mathrm{l}(s - A(N))$ contains a non-zerodivisor c. Let $c = a + x$, with $a \in a - A(N)$, $x \in \mathrm{l}(s - A(N))$. We set $K =$ $= s - A(N)$. Now $Kc \supseteq Kc^2 \supseteq \ldots$. Since K is with dcc on its left N-subgroups, and each Kc^s $(s \in \mathbb{Z}^+)$ is a left N-subgroup of K, we have a $t \in \mathbb{Z}^+$ with $Kc^t = Kc^{t+1}$. Then for $\alpha \in K$ we get $\beta \in K$, $\alpha = \beta c$. So $K = KC$ and it gives $a = ec$, for some $e \in K$. Now $c = a + x = ec + x$, $e = \sum s_i$, s_i distributive then $ec = e^2c$ gives $e = e^2$ and for any $b \in K$, $b = be$. Thus $K \subseteq Ne$ and also $Ne \subseteq K$ gives $K = Ne$. ◊

Finally we mention our last

Note. When N is strongly regular as in [6], $Ne = eN$ and thus $s-A(N)$ is a direct summand of N.

References

[1] BARUA, N. M.: Near-rings and near-ring modules, some special types, Naya Prakash, Calcutta, 1984.

[2] CHATTERS, A. W. and HAZARNAVIS, C. R.: Rings with chain conditions, Pitman Advanced Publishing Program, 1980.

[3] CHOWDHURY, K. C. and MASUM, A.: A note on regular left Goldie near-rings, *Nat. Acad. Sci. Letters* **12** (1989), 433–435.

[4] CLAY, J. R.: The near-rings of low order, *Math. Zeitschrift* **104** (1968), 364–371.

[5] GOLDIE, A. W.: Non commutative principal ideal rings, *Arch. Math.* (Basel) **13** (1962), 214–221.

[6] JAYARAM, C. and JOHNSON RAJKUMAR, L.: Strongly regular near-rings, *J. of the Indian Math. Soc.* **5** (1989), 433–435.

[7] OSWALD, A.: Near-rings in which every N-subgroup is principal, *Proc. London Math. Soc.* **(3) 23** (1974), 67–88.

[8] OSWALD, A.: Near-rings with chain conditions on right annihilators, *Proc. Edinburgh Math. Soc.* **23** (1980), 123-127.

[9] PILZ, G.: Near-rings, North Holland, Amsterdam, 1977.

[10] TIWARI, K. and SETH, V.: Clasical near-ring of left and right quotients, *Progress Math.* **12** (1978), 115–123.