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# M–BLOCKS OF SOLVABLE GROUPS

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**Abstract**: We give sufficient conditions for a $p$-block of a solvable group to be an $M$-block. We also investigate the question how the principal block or the blocks of maximal defect determine monomiality.

**1.** The notion of $M$-blocks was introduced by C. Bessenrodt in [1]: an $M$-block is a $p$-block, in which each irreducible ordinary character is monomial. In [1] we can find sufficient conditions to guarantee that a $p$-block is an $M$-block. In this note first we give an equivalent formulation of this theorem for groups of odd order, using the notion of $(p, q)$-groups. We also prove the odd order form of the theorem using weakened conditions. Furthermore, we give a sufficient condition for a $p$-block to be an $M$-block in the case when the group is $p$-nilpotent.

We show that the fact that the characters in the principal block are monomial or subnormally monomial (SM) does not imply in general that these properties would hold for all irreducible characters: one can find counterexamples even among Frobenius groups, if the prime $p$ does

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not divide the order of the kernel. If $p$ is a divisor of the order of the kernel, then monomiality (or subnormal monomiality) of characters in the principal block implies these properties for all the characters.

We prove in certain special cases that in a finite solvable group $G$, the monomiality of characters of all $p$-blocks of maximal defect implies monomiality for the whole group. In fact this is true

- if all proper subgroups and all proper homomorphic images of $G$ are $M$-groups;
- if the $p$-blocks of maximal defect of every proper subgroup of $G$ are $M$-blocks.

Unfortunately this property does not hold in general. We also give an example to show that the analogous statement for $SM$ is not true either; namely it can happen that every block of maximal defect is an $SM$-block, but the group is not an $SM$-group.

All groups in this paper are assumed to be finite and solvable. Unless otherwise stated, characters are ordinary characters over an algebraically closed field $K$ of characteristic 0, and we assume that $K$ is the field of quotients of a complete discrete valuation ring $R$ with residue class field $F$, giving the $p$-modular system $(K, R, F)$. For basic definitions and notations the reader is referred to [4], [11] and [12].

**2.** Let us recall the above mentioned result of [1]:

**Theorem** (Bessenrodt). *Let $G$ be a finite solvable group and suppose that $G$ has a normal subgroup $N$ such that:*

(i) *all chief sections of $G/N$ have odd degree;*

(ii) *$N$ has quaternion-free, respectively modular, Sylow $q$-subgroups, for all $q \neq p$.*

*Then any p-block of $G$ with quaternion-free, respectively modular, defect group is an $M$-block.*

Let $G$ be a group of odd order. Assume that $p$ is a prime divisor of $|G|$, such that the order of $p$ mod $q$ is odd for every other prime divisor $q$ of $|G|$. Then, according to [16], each $p$-chief factor of $G$ is an odd dimensional vector space. Combining this result with Bessenrodt's theorem we get:

**Proposition 2.1.** *Let $G$ be a finite solvable group of odd order and $p$ a prime divisor of $|G|$. Let $N \triangleleft G$ be such that:*

(i) *for each pair $r$ and $q$ of distinct prime divisors of $|G/N|$, the* *order of r mod q is odd;*

(ii) *for each prime divisor q of $|N|$, if $q \neq p$ then N has modular Sylow q-subgroups.*

*Then every p-block of G with modular defect group is an M-block.*

According to [16], for those prime divisors $r$ of $|G|$, whose order mod $q$ is odd for each other prime divisor $q$, the $r$-chief factors are odd dimensional vector spaces. Thus if we want to formulate a theorem similar to that of Bessenrodt for the case when $|G|$ is odd, then we need to require only that for those prime divisors $r$ of $|G/N|$, whose order mod $q$ is even for some other prime divisor $q$ of $G/N$, the $r$-chief sections of $G/N$ should be odd dimensional vector spaces. So we can state the following:

**Proposition 2.2.** *Let G be a finite solvable group of odd order, and let p be a prime divisor of $|G|$. Let $N \triangleleft G$ be such that:*

(i) *for each pair r and q of distinct prime divisors of $|G/N|$, for which the order of r mod q is even, the r-chief factors of every subgroup of $G/N$ are odd dimensional vector spaces;*

(ii) *N has modular Sylow q-subgroups for every prime divisor q of $|N|$ different from p.*

*Then every p-block of G with modular defect group is an M-block.*

Let $(p, q)$ denote a minimal non-$p$-nilpotent group as it is described by a theorem of Ito in [9]. (For the notation see [2].) Then $(p, q)$ has a normal Sylow $p$-subgroup $P$, which is special, and a cyclic Sylow $q$-subgroup, whose $q$-th power is in the center of the group. Let us recall that in [2] $G \geq (p, q)$ meant that $G$ contains such a subgroup, so $G$ is not $p$-nilpotent. Otherwise we write $G \not\geq (p, q)$. In [2] it was shown that if $G \not\geq (p, q)$, then this property is inherited by homomorphic images.

For showing the connection between $p$-chief factors and $(p, q)$-groups we prove the following lemma, which was stated already in [3].

**Lemma 2.3.** *Let G be a solvable group of odd order and let p be a prime divisor of $|G|$. Then each p-chief factor of every subgroup of G is an odd dimensional vector space over $GF(p)$ if and only if $G \not\geq (p, q)$ for all prime divisors q of $|G|$ for which the order of p mod q is even.*

**Proof.** Let us suppose first that each $p$-chief factor of every subgroup of $G$ is odd dimensional. If there would be such a $(p, q)$-subgroup $U$ in $G$ then $U = PQ$, where $P \triangleleft U$, $Q =< x >$ and $x^q \in Z(U)$, $P$ is special and $< x > / < x^q >$ acts faithfully and irreducibly on $P/P'$. Then by Satz 3.10 in Chapter II of [9], $\dim_{GF(p)} P/P' = o(p) \bmod q$, which is, by assumption, even. This contradicts the assumption on the dimension of $p$-chief factors of subgroups of $G$.

Let us suppose now that $G \not\geq (p,q)$ for all prime divisors $q$ of $|G|$ for which the order of $p$ mod $q$ is even. We prove that the $p$-chief factors of the subgroups of $G$ are odd dimensional. Let us assume that $G$ is a counterexample of minimal order. Then the statement is true for every proper subgroup of $G$. By [2] the conditions $G \not\geq (p,q)$ are inherited by factors, so by induction the statement is true for every proper factor of $G$. Thus we may assume that the even dimensional $p$-chief factor of $G$ is a minimal normal subgroup $L$ of $G$. Let $S$ be a Sylow $q$-subgroup of $G$ for a prime $q$ such that the order of $p$ mod $q$ is even. Then $S \leq C_G(L)$, otherwise $(p,q) \leq LS$ would hold. Let $\overline{G} = G/C_G(L)$. Then all the prime divisors $q \neq p$ of $|\overline{G}|$ have the property that the order of $p$ mod $q$ is odd. As $\overline{G}$ is contained in the group of automorphisms of $L$ we can form the semidirect product of $L$ by $\overline{G}$. For the prime divisors $q \neq p$ of this group we also have that the order of $p$ mod $q$ is odd. So, by [16], all $p$-chief factors of $\overline{G}K$ are odd dimensional, which contradicts the assumptions on $L$. $\Diamond$

We can now reformulate the assumption (i) in Prop. 2.2 using the result of the above lemma: we will require that for all prime divisors $r \neq q$ of $|G/N|$ for which the order of $r$ mod $q$ is even, $G/N \not\geq (r,q)$.

**Proposition 2.4.** *Let $G$ be a finite solvable group of odd order, and $p$ a prime divisor of $|G|$. Let $N \lhd G$ be such that*

(i) *for each pair of distinct prime divisors $r$ and $q$ of $|G/N|$ for which the order of $r$ mod $q$ is even, $G/N \not\geq (r,q)$;*
(ii) *$N$ has modular Sylow $q$-subgroups for every prime divisor $q$ of $|N|$ different from $p$.*

*Then every $p$-block of $G$ with modular defect group is an $M$-block.*

We will prove below that we can also weaken the assumption in (i), assuming it only for primes $r \neq p$.

**Theorem 2.5.** *Let $G$ be a finite solvable group of odd order and $p$ a prime divisor of $|G|$. Let $N \lhd G$ be such that:*

(i) *for each pair of distinct prime divisors $r$ and $q$ of $G/N$ such that $r \neq p$ and the order of $r$ mod $q$ is even, $G/N \not\geq (r,q)$;*
(ii) *$N$ has modular Sylow $q$-subgroups for all prime divisors $q$ of $|N|$ different from $p$.*

*Then every $p$-block of $G$ with modular defect group is an $M$-block.*

**Proof.** Let us take a block $B$ of $G$ with modular defect group and let $\chi \in \mathrm{Irr}(B)$ be an irreducible character. Similarly to the proof of Th. 2.12 in [1], one can assume by Fong Reduction that the defect group of the block $B$ is a Sylow $p$-subgroup of $G$. Since the assumptions are inherited by subgroups and homomorphic images, we may assume that $\chi$ is faithful and primitive. As in the proof of Theorem 2.12 in [1], we can show that $N$ is abelian.

We prove by induction that if for a group $G$ the Sylow $p$-subgroup is modular, $N \triangleleft G$ is abelian, and (i) holds for $G/N$, then $G$ is monomial. Otherwise let $\chi \in \mathrm{Irr}(G)$ be a non-monomial character. By induction $\chi$ is faithful and primitive and $G$ is a minimal non-$M$-group. According to the characterization by Price (see [14]), $G = \mathrm{Fit}(G)A$, where $\mathrm{Fit}(G)$ is an extraspecial $r$-group of exponent $r$ for some prime $r$, $A$ is a cyclic group of order $q$ for some prime, and $\mathrm{Fit}(G)/Z(G)$ is a faithful irreducible $A$-module. So by Satz 3.10 in Chapter II of [9], $\dim_{GF(r)} \mathrm{Fit}(G)/Z(G) = o(r) \bmod q$, which is even, since the factor is a nonsingular symplectic $G$-module. Here $r \neq p$, because of the assumption on the Sylow $p$-subgroup of $G$. As $G/Z(G)$ is not $r$-nilpotent, $G/Z(G) \geq (r, q)$. Since $N$ is abelian, it is contained in $Z(G)$, so $G/N \geq (r, q)$, too. This contradicts the assumptions of the second part of the proof. But then we get that the group $G$ in the first part of the proof is monomial. $\Diamond$

**3.** Some properties of groups can be checked already using characters of the principal block or blocks of maximal defect, see for example [13]. In this section we shall investigate how the behaviour of blocks of maximal defect or that of the principal block determines whether the group is monomial or subnormally monomial.

It is not true that if all characters of the principal block are monomial then the group is a monomial group. For example, if $G = SL(2,3)$, for $p = 3$ the principal block, $B_0(G)$ consists of three linear characters, so it is an $M$-block, but $G$ is not an $M$-group. In general, for every $p$-nilpotent group $G$ we have that $B_0(G) = \mathrm{Irr}\left(G/O_{p'}(G)\right)$ is an $M$-block, as all $p$-groups are monomial. So if $G$ itself is not an $M$-group then we again have a counterexample. Moreover, the following result is true.

**Proposition 3.1.** *Let $G$ be a solvable $p$-nilpotent group. Then every* *p-block containing a linear character is an M-block. If $G/O_{p'}(G)$ is abelian, then every block containing a monomial character is an M-block, and every block containing a subnormally monomial character contains only SM-characters, in other words, it is an SM-block.*

**Proof.** Let $H = O_{p'}(G)$, $B$ a block of $G$ and $\lambda \in \mathrm{Irr}(B)$ a linear character. So ${\lambda_H}^G$ is the only projective indecomposable character in $B$. According to Gallagher's Theorem (see [11]), ${\lambda_H}^G = \sum \beta(1)(\beta\lambda)$, where $\beta$ is running through $\mathrm{Irr}(G/H)$. Since $G/H$ is a $p$-group, each $\beta$ is monomial. As all $\chi \in \mathrm{Irr}(B)$ occur as constituents of ${\lambda_H}^G$, so we have that $\chi$ has the form $\beta\lambda = (\gamma_{L/H}\lambda_L)^G$ for some subgroup $L$ and linear character $\gamma$. Thus $B$ is an $M$-block.

Let $G/H$ now be abelian. Then if $\chi \in \mathrm{Irr}(B)$ and $(\chi_H, \phi) \neq 0$, then for each other $\psi \in \mathrm{Irr}(B)$ we have $(\psi_H, \phi) \neq 0$, too. By [15], $\psi = \lambda\chi$ for some linear character $\lambda \in \mathrm{Irr}(G/H)$. Thus, if $\chi$ is monomial, then so is $\psi$. If $\chi$ is $SM$ then so is $\psi$. $\Diamond$

**Remark 3.2.** The above statement is not true for arbitrary groups. For $G = SL(2,3)$ and $p = 2$ we have $O_{p'}(G) = 1$, so $B_0(G) = \mathrm{Irr}(G)$. Thus the principal block is not an $M$-block. Also, it is easy to see that in general an $M$-block in a $p$-nilpotent group does not necessarily contain a linear character. This is shown for $p = 2$ by the following example.

**Example 3.3.** Take the extraspecial group of order $3^3$ and exponent 3, extended by an order 2 automorphism, which acts on its commutator factorgroup reducibly. For $p = 2$ the principal block contains only linear characters, so it is an $SM$-block, but this group is not subnormally monomial. However it is supersolvable, so it is an $M$-group. This group is 2-nilpotent. It has defect zero $M$-characters of degree 2, so their blocks are $M$-blocks which do not contain any linear character.

In the special case, when $G$ is a Frobenius group the behaviour of the principal block sometimes determines whether the group is monomial (or subnormally monomial):

**Theorem 3.4.** *Let $G$ be a solvable Frobenius group with kernel $N$, and let $p$ be a prime divisor of $|G|$. If $p$ is a divisor of $|N|$, then the following statements are equivalent:*

(i) *$B_0(G)$ is an M-block.*
(ii) *$G$ is a monomial group.*
(iii) *$G$ is a subnormally monomial group.*

(iv) $B_0(G)$ *is a block containing only subnormally monomial characters.*

**Proof.** (i) $\Longrightarrow$ (ii). Let $\chi \in \mathrm{Irr}(G)$. If $\mathrm{Ker}(\chi) \not\geq N$ then $\chi = \phi^G$ for some $\phi \in \mathrm{Irr}(N)$, so $\chi$ is monomial, since $N$ is nilpotent. If $\mathrm{Ker}(\chi) \geq$ $\geq N$ then $(\chi_N, 1_N) \neq 0$. But $1_N \in B_0(N)$. Let $P$ be a defect group of $B_0(N)$. As $C_G(P) \leq N$, it follows from Brauer's first main theorem that $B_0(N)^G = B_0(G)$, and that $B_0(G)$ is the only block of $G$ which covers $B_0(N)$. So $\chi \in B_0(G)$ and thus, $\chi$ is a monomial character.

The implication (ii) $\Longrightarrow$ (iii) follows easily (see [7]), while (iii) $\Longrightarrow$ $\Longrightarrow$ (iv) and (iv) $\Longrightarrow$ (i) are trivial. ◊

**Example 3.5.** Let $G$ be a Frobenius group with kernel $N$ and complement $H$ and let us assume that the prime $p$ does not divide $|N|$. Then it may happen that $B_0(G)$ is an $M$-block, but $G$ is not monomial. Namely, if $p$ does not divide $|N|$, then the $p$-blocks of $G$ are the $p$-blocks of $H$, together with those blocks of $G$ of zero defect, which contain exactly one irreducible ordinary character of $G$ not containing $N$ in its kernel. So if we take a Frobenius group $G$ with complement $SL(2,3)$, then for $p = 3$ we shall get that $B_0(G)$ is an $M$-block, but $G$ is not an $M$-group. *Proof:* If $\chi \in \mathrm{Irr}(G)$ and $\mathrm{Ker}\,\chi \not\geq N$, then $\chi = \zeta^G$ for some $\zeta \in \mathrm{Irr}(N)$. So $p$ divides $\chi(1)$ and $\chi$ is of defect zero, so it is the single irreducible ordinary character in its block. If $\chi \in \mathrm{Irr}(G)$ and $\mathrm{Ker}\,\chi \geq N$, then let us suppose that $\chi$ belongs to a $p$-block $B$ of $G$. Since $(|N|, p) = 1$, $\mathrm{Irr}\left(B_0(N)\right) = \{1_N\}$, and since $B$ covers $B_0(N)$, for each $\psi \in \mathrm{Irr}(B)$, we have that $\mathrm{Ker}\,\psi \geq N$. We claim that $B_1 = \{\chi_H | \chi \in \mathrm{Irr}(B)\}$ gives a block of $H$. Let $\phi, \psi \in B$. Then

$$\frac{\phi(h)|G : C_G(h)|}{\phi(1)} \equiv \frac{\psi(h)|G : C_G(h)|}{\psi(1)} \pmod{p}.$$

Since $C_G(h) \leq H$, $|G : C_G(h)| = |G : H||H : C_H(h)|$ and since $(|G : H|, p) = 1$, we get

$$\frac{\phi(h)|H : C_H(h)|}{\phi(1)} \equiv \frac{\psi(h)|H : C_H(h)|}{\psi(1)} \pmod{p}.$$

This means that $\phi$ and $\psi$ belong to the same $p$-block $b$ of $H$. So $B_1 \subseteq b$. Let $x$ and $y$ be a $p$-regular and a $p$-singular element of $H$, respectively. Then we have $\sum_{\chi \in B_1} \chi(x)\overline{\chi(y)} = \sum_{\chi \in B} \chi(x)\overline{\chi(y)} = 0$, by the block ortogonality relation. By the theorem of Osima (see e.g. [12]), $B_1$ is a union of blocks of $H$, so $B_1 = b$. In particular, we get that $\chi \in B_0(G)$ if and only if $\chi_H \in B_0(H)$. In this case, if $B_0(H)$ is an $M$-block and $H$

is non-monomial, then $B_0(G)$ is an $M$-block and $G$ is non-monomial. Thus, if we take a Frobenius group $G$ with complement $SL(2,3)$, then for $p=3$, $B_0(G)$ is an $M$-block, but $G$ is not an $M$-group. ◊

Thus, in order to guarantee monomiality, we need to assume more. In some special cases we can prove that if in a group $G$ each $p$-block of maximal defect is an $M$-block, then the group is an $M$-group.

**Proposition 3.6.** *Let $G$ be a solvable group for which every $p$-block of maximal defect of every subgroup is an $M$-block. Then $G$ is a subgroup-closed $M$-group.*

**Proof.** We shall prove the statement by induction on the order of $G$. Let us suppose that $G$ is a group satisfying the assumptions, and assume that we already have proved the statement for groups of smaller size. Thus, in particular, every proper subgroup of $G$ is an $M$-group. By Th. 1.5 in Chapter X of [4], if $G$ is $p$-solvable then $G$ has one block if and only if $O_{p'}(G)=1$. Thus, if $O_{p'}(G)=1$ then the statement holds trivially.

Assume now that $O_{p'}(G)\neq 1$. Let $B$ be a block of non-maximal defect and let $\chi\in \mathrm{Irr}(G)$ be a character belonging to the block $B$. Then, by Clifford's Theorem, $\chi_{O_{p'}(G)} = e\sum\beta_i$. If the inertiagroup $T=I_G(\beta)<G$, where $\beta=\beta_1$, then $\chi$ is induced from a character of $T$. By induction, $T$ is monomial, so $\chi$ is also monomial.

If, on the other hand, $T=G$, then we use Fong Reduction, as it is described in Section 1. of [6]. According to Lemma 1A in [6], every block of the representation group $H$ of $B$ has maximal defect. Note that $G/O_{p'}(G)\simeq H/O_{p'}(H)$, and if $B''$ is the block of $H$ corresponding to $B$ then their defect groups are isomorphic. Thus, we get that $B$ also has maximal defect, contradicting our assumption. ◊

Next, we prove that if every proper subgroup and every proper section of a solvable group $G$ is an $M$-group and each block of maximal defect of $G$ is an $M$-block, then $G$ is an $M$-group, too.

**Proposition 3.7.** *Let $G$ be a solvable minimal non-$M$-group. Then $G$ contains a block of maximal defect which is not an $M$-block.*

**Proof.** Let us suppose that, contary to our statement, $G$ is a solvable minimal non-$M$-group and every block of maximal defect in $G$ is an $M$-block. Then $O_{p'}(G)>1$, as otherwise there would be only one block, and then $G$ would be an $M$-group. Let $N=O_{p'}(G)$. Let $\chi\in$ $\in \mathrm{Irr}(G)$ be a non-monomial character belonging to a block $B$. Then, if $\beta\in\mathrm{Irr}(N)$ is a constituent of $\chi_N$, then its inertia group could not be smaller than $G$, as otherwise $\chi$ would be induced from a proper subgroup, which is an $M$-group. Thus, $\beta$ is invariant in $G$. But then, similarly to the argument above, we would get, by Fong Reduction, that $B$ is of maximal defect, and this is a contradiction. $\diamondsuit$

Unfortunately, in general the monomiality of blocks of maximal defect does not guarantee monomiality. The following example is due to Professor Howlett.

**Example 3.8.** Let $G = ED$ be the semidirect product of an extraspecial group $E$ of order $5^3$ and exponent 5 and of the dihedral group $D$ of order 6 acting on $E$ faithfully, with the property that its involutions are inverting $Z(E)$. Each 2-block of maximal defect of $G$ is an $M$-block. However the degree 10 characters in $\mathrm{Irr}(G)$ are not monomial.

*Proof:* Professor Howlett shows that if $\chi$ belongs to a block of maximal defect, then its kernel contains $Z(E)$. First of all, $\chi(1)$ is odd. If $\lambda$ is an irreducible constituent of $\chi_{Z(E)}$, then each involution $t \in D$ is contained in the inertiagroup $I_G(\lambda)$, as otherwise $\chi$ could be induced from a subgroup of index divisible by 2, and this is impossible. Let $z \in$ $\in Z(E)$. Then $\lambda(tzt^{-1}) = \lambda(z^{-1})$, so $1 = \lambda\big([t, z^{-1}]\big) = \lambda\big(z^{-2}\big)$. Hence $z^{-2} \in \mathrm{Ker}\,\chi$ and thus $Z(E) \leq \mathrm{Ker}\,\chi$. Since $G/Z(E)$ has abelian Sylow subgroups, $\chi$ is monomial. As $G$ does not contain a subgroup of index 10, the degree 10 characters in $\mathrm{Irr}(G)$ are not monomial.$\diamondsuit$

*Another possibility for the proof* is to construct such a group, and use the functions of the GAP system to check monomiality. The following construction is due to Balázs Szegedy. Consider the group $G$ with the following presentation:

$$G = \langle a, b, x, t a^5 = b^5 = [a,b]^5 = 1,\ x^3 = t^2 = 1,\ x^t = x^{-1},$$
$$a^t = a^4,\ b^t = b,\ a^x = a^2 b,\ b^x = a^3 b^2,\ [a,b]^t = [a,b]^{-1}\rangle.$$

One can easily check, using the GAP system (cf. [16]), that this group has the required properties. One defines it as an `FpGroup` with the above generators and relators. With the function `LowIndexSubgroupFpGroup`, one can determine that there is no subgroup of index 10 in it, so the degree 10 irreducible characters of this group are not monomial. With the function `OperationCosetsFpGroup`, one makes a permutationgroup from this group, and then with the function `AgGroup`, one transforms it into an `AgGroup`. In this form the function `CharTable` works very quickly, and we get the charactertable of our group. With the function `PrimeBlocks` one determines the 2-blocks, and their defects. It turns out that the blocks of maximal defect are: the principal block, containing two linear characters, and four other blocks, containing two degree

3 characters each. Then one applies `TestMonomialQuick`, to prove that these characters are monomial.

**Example 3.9.** There exists a monomial, but not subnormally monomial group such that for some $p$ each $p$-block of maximal defect consists of $SM$ characters. Namely, there exists a supersolvable, and thus monomial group which is not $SM$, but each block of maximal defect is an $SM$-block. Take the Borel subgroup of $GL(3,3)$ and $p = 2$. Its blocks of maximal defect contain only linear characters, so they are $SM$-blocks, however it has characters of degree 6 which are not $SM$.
*Proof:* The statement can be easily checked by GAP, using the function `TestSubnormallyMonomial`. ◊

**Remark 3.10.** As Ex. 3.8 and Prop. 3.6 show, the property of having only $M$-blocks as blocks of maximal defect in a group is not inherited by subgroups. The property that each block of maximal defect is an $SM$-block is not inherited by subgroups either: the group described in Ex. 3.3 is a subgroup of the group described in Ex. 3.9, and it has 2-blocks of maximal defect, which are not $SM$.

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