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ACCELERATION PROPERTIES OF ROBOT-MANIPULATORS

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Abstract: The motion of a p-parametrical robot-manipulator is expressed, its velocity and acceleration operators are found. In particular, three parametrical robot-manipulators with only rotational axes are considered. All robot-manipulators of this type, which have one component of Coriolis acceleration equal to zero at each position are found.

1. Introduction

In this paper we shall study the motion of robot-manipulators. We express the axes of a robot-manipulator using Plücker coordinates and we shall calculate the velocity operator and the acceleration operator for instantaneous position of robot-manipulator.

Let us have the Euclidean space E_3 with a system of Cartesian coordinates (x_1, x_2, x_3). Let $P = A + \lambda\bar{x}$ be a straight line in E_3. Plücker coordinates $\chi = (\bar{x}; \bar{y})$ of the straight line P consist of the pair $(\bar{x}; \bar{y})$ where $\bar{x}$ is the unit vector of P and $\bar{y} = A \times \bar{x}$.

Let $r(\varphi)$ be a matrix of the revolution around the line $\chi = A + \lambda\bar{x}$ where φ is the angle of revolution. The expression of this revolution is

$$r(\varphi) = \begin{pmatrix} 1 & 0 \\ t & \gamma \end{pmatrix}$$

where γ is an orthogonal matrix and t is a column matrix. For the derivative of $r(\varphi)$ we have $r'(\varphi) = r'(0)r(\varphi)$ and we can write the matrix $r'(0)$ in the following form

$$r'(0) = \begin{pmatrix} 0 & 0 \\ t' & \gamma' \end{pmatrix}$$

where $t'(0) = \bar{y}$, $\gamma'(0) = \bar{x}$. Here we have used the following identification

$$\gamma'(0) = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} \Longleftrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \bar{x}.$$

The matrix $r'(0)$ can be identified with Plücker coordinates of the axis of revolution $r(\varphi)$ and we have $r'(0) = \chi = (\bar{x}, \bar{y})$.

A *robot-manipulator* with p-degrees of freedom is a product of p-revolutions around p-axes $\chi_1, \chi_2, \dots \chi_p$ which are given by their Plücker coordinates $X_i = (\bar{x}_i; \bar{y}_i)$. The motion of the end-effector of this robot-manipulator is expressed by the matrix $g(\varphi_1, \varphi_2; \dots; \varphi_p) = r_1(\varphi_1) \cdot r_2(\varphi_2) \dots r_p(\varphi_p)$, where $r_i(\varphi_1)$ is the matrix of the revolution around χ_i. If $\varphi_i = \varphi_i(t)$ are functions of time t we obtain a one-parametric motion of the end-effector $g(t) = r_1(\varphi_1(t)) \cdot r_2(\varphi_2(t)) \dots r_p(\varphi_p(t))$. Trajectory of a point $\bar{A}$ of the end-effector is $A(\varphi_1, \varphi_2 \dots \varphi_p) = g(t)\bar{A}$. See [2].

2. Velocity and acceleration operators for robot-manipulators

Let Ω be the *velocity operator* of $g(t)$, θ be the *acceleration operator* of $g(t)$. This means that $v_A = \Omega A$, $a_A = \theta A$, where v_A is the velocity of the trajectory of $\bar{A}$ at A and a_A is the acceleration of the trajectory of $\bar{A}$ at A. Computation yields $\Omega = g'g^{-1}$, $\theta = \Omega' + \Omega^2$. If we express Ω for a p-parametrical robot-manipulator we obtain $\Omega = \sum_{i=1}^{p} Y_i v_i$ where Y_i is the instantaneous position of i-th axis and v_i is the angular velocity of $r_i(\varphi_i(t))$; $v_i = \frac{d\varphi_i}{dt}$. For the derivative of Ω we have

$$\Omega' = \Big(\sum_{i=1}^{p} Y_i \cdot v_i\Big)' = \sum_{i=1}^{p} Y_i' \cdot v_i + \sum_{i=1}^{p} Y_i \frac{dv_i}{dt}.$$

We can split the acceleration operator into three parts:

(1) Ω^2 is the centrifugal acceleration;

(2) $\sum_{i=1}^{p} Y_i \varepsilon_i$ is the Euler acceleration where $\varepsilon = \frac{dv_i}{dt}$ is the angular

acceleration of $r_i(\varphi_i(t))$;

(3) $\sum_{i=1}^{p} Y_i' v_i = \sum_{i<j=1}^{p} Y_i \times Y_j v_i v_j$ is the Coriolis acceleration where $Y_i \times Y_j$ is the cross-product of Plücker coordinates of Y_i with Y_j. See [1].

Let us consider the *Coriolis acceleration* for 3-parametric robot-manipulators with rotational axes X_1, X_2, X_3. Let the instantaneous position of these axes be Y_1, Y_2, Y_3. Then the Coriolis acceleration C is

$$C = Y_1 \times Y_2 v_1 v_2 + Y_1 \times Y_3 v_1 v_3 + Y_2 \times Y_3 v_2 v_3.$$

Let

$$Y_1 = (\bar{x}'; \bar{y}'),\ Y_2 = (\bar{x}'', \bar{y}''),\ Y_3 = (\bar{x}''', \bar{y}''').$$

We define $\langle Y_1, Y_2 \rangle = x'y'' + y'x''$ as a scalar product in the 6-dimensional space of Plücker coordinates. Let V_1 denote the vector space generated by Y_1, Y_2, Y_3 and V_2 be its orthogonal complement. It follows that the Coriolis acceleration has two components: C_1 into V_1, C_2 into V_2.

Let us find all 3-parametrical robot-manipulators with the component $C_1 = 0$ at all positions. Y_1, Y_2, Y_3 is the base of V_1 and therefore we must have

(1) $$\langle Y_1, C \rangle = 0,\ \ \langle Y_2, C \rangle = 0,\ \ \langle Y_3, C \rangle = 0.$$

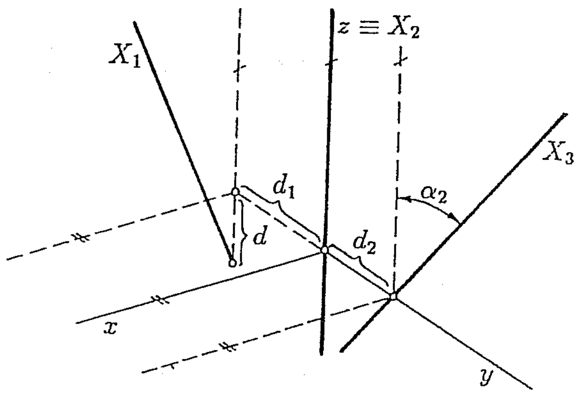

Fig. 1

(1) leads to the equation $\langle Y_1, Y_2 \times Y_3 \rangle = 0$. We shall express this condition in coordinates. Let us choose the coordinate system (see Fig. 1) in the fixed space. The Plücker coordinates of the axes X_1, X_2, X_3 are:

$$X_1 = \begin{pmatrix} -\sin\alpha_1 & d_1\cos\alpha_1 \\ 0 & d\sin\alpha_1 \\ \cos\alpha_1 & d_1\sin\alpha_1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix},$$

$$X_3 = \begin{pmatrix} -\sin\alpha_2 & d_2\cos\alpha_2 \\ 0 & 0 \\ \cos\alpha_2 & d_2\sin\alpha_2 \end{pmatrix}.$$

For the instantaneous axes Y_1, Y_2, Y_3 we obtain $Y_1 = X_1$, $Y_2 = X_2$,

$$Y_3 = \begin{pmatrix} -\cos\varphi_2\sin\alpha_2 & d_2\cos\varphi_2\cos\alpha_2 \\ -\sin\varphi_2\sin\alpha_2 & d_2\sin\varphi_2\cos\alpha_2 \\ \cos\alpha_2 & d_2\sin\alpha_2 \end{pmatrix}.$$

$$\frac{\partial Y_3}{\partial\varphi_2} = Y_2 \times Y_3, \quad \frac{\partial\langle Y_1, Y_3\rangle}{\partial\varphi_2} = \langle Y_1, Y_2 \times Y_3\rangle = 0.$$

This shows that $\langle Y_1, Y_3\rangle$ must be constant with respect to φ_2. Computation yields

$$\langle Y_1, Y_3\rangle = -d_2\sin\alpha_1\cos\varphi_2\cos\alpha_2 + d_2\cos\alpha_1\sin\alpha_2 -$$
$$-d_1\cos\alpha_1\cos\varphi_2\sin\alpha_2 + d\sin\alpha_1\sin\varphi_2\sin\alpha_2 + d_1\sin\alpha_1\cos\alpha_2.$$

The coefficients by $\cos\varphi_2$, $\sin\varphi_2$ must be zero and therefore we have the following conditions

(2) $\quad d\sin\alpha_1\sin\alpha_2 = 0, \quad d_2\sin\alpha_1\cos\alpha_2 + d_1\cos\alpha_1\sin\alpha_2 = 0.$

There are three possibilities for the axes X_1, X_2, X_3 of a 3-parametrical robot-manipulator:

(a) If $\sin\alpha_1 = 0$, then $d_1\cos\alpha_1\sin\alpha_2 = 0$. From $d_1 = 0$ follows that the axes X_1, X_2 coincide.

(b) From $\sin\alpha_2 = 0$ follows that the axes X_1, X_2, X_3 are parallel.

(c) If $\sin\alpha_1\sin\alpha_2 \neq 0$ then $d = 0$.
From (2) we get

(3) $$d_2\cot\alpha_2 + d_1\cot\alpha_1 = 0.$$

It means that the axes X_1, X_2, X_3 have a common perpendicular and satisfy the condition (3). From these considerations we see the following

Theorem 3. *3-parametrical robot-manipulators with three rotational axes X_1, X_2, X_3 have the component C_1 of the Coriolis acceleration equal to zero in the following cases:*

(1) *the axes X_1, X_2 or X_2, X_3 coincide;*

(2) *the axes X_1, X_2, X_3 are parallel;*

(3) *the axes X_1, X_2, X_3 have a common perpendicular and satisy* (3).

Remark. Let X_1, X_2, X_3 be axes of the robot-manipulator from the case (c). Let us denote by $\bar{X}_3$ the straight line defined analogically to X_3

with the angle α_2 substituted by $-\alpha_2$. Then $X_1, \bar{X}_3$ are conjugated polars of the linear complex with axis X_2 and parameter $v_0 = d_1 \tan \alpha_1 = -d_2 \tan \alpha_2$.

References

[1] CRAIG, J. J.: Introduction to Robotics. Mechanics and Control, Addison–Wesley Publishing Company, 1986.

[2] KARGER, A. and NOVÁK, J.: Space kinematics and Lie groups, Gordon and Breach. New York–London, 1985.