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w-JORDAN NEAR-RINGS I

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Abstract: Let N be a zero-symmetric near-ring with an invariant series whose factors are N-simple. We prove that the radical $J_2\ (N)$ is nilpotent and the factor $N/J_2\ (N)$ is a direct sum of a finite number of A-simple and strongly monogenic near-rings. Moreover we characterize nilpotent near-rings with invariant series whose factors are of prime order.

Introduction and general results

Many authors have studied near-rings containing particular chains of ideals (see [5,8,10]) and have often shown the existence of links between these chains of ideals and the structure of the near-rings under consideration. In this paper we begin a study of near-rings with an invariant series whose factors belong to certain given classes. In particular we study here the zero-symmetric case; the general case and the construction of finite near-rings satisfying these conditions will be covered in future papers.

For the zero-symmetric near-rings with an invariant series whose factors are N-simple, we obtain a result analogous to the Artin-Noether theorem. We prove that a zero-symmetric near-ring N with an invariant series whose factors are N-simple has the radical $J_2(N)$ nilpotent and the factor $N/J_2(N)$ is a direct sum of A-simple and strongly monogenic near-rings. Moreover we discuss the finite case and characterize the near-rings with an invariant series whose factors are of prime order. We prove a necessary and sufficient condition so that N is nilpotent and we establish a link between the nilpotence index and the length of the series. In the case in which index and length coincide, we prove that the order of N is a prime power.

In the following we will often refer to [12] without express recall.

Let N be a left near-ring. A finite system of subnear-rings of N contained in one another

$$N = N_1 \supset N_2 \supset \ldots \supset N_n = \{0\}$$

is called a *normal series* of N if every subnear-ring N_i, $i \in \{1, 2, \ldots, n\}$, is a proper ideal in N_{i-1}, an *invariant series* of N if every subnear-ring N_i, $i \in \{1, 2, \ldots, n\}$, is a proper ideal of N. The factor-near-rings N_i/N_{i+1} are called *principal factors* of the invariant series. For invariant series, in the following, we will indicate $N_i/N_{i+1}, N_i/N_{i+2}, \ldots, N_i/N_{i+k}$ respectively with $N_i', N_i'' \ldots, N_i^k$ and with $f_i', f_i'', \ldots, f_i^k$ the corresponding canonical epimorphisms.

Let us consider now the following classes of near-rings:

S_0: class of simple near-rings ;
S_1: class of simple and strongly monogenic near-rings ;
S_2: class of N_0 -simple near-rings(1);
S_3: class of near-rings without proper subnear-rings;
S_4: class of near-rings of prime order.

Definition 1. A near-ring N is a *w-Jordan near-ring* (*wJ-near-ring*) if it has an invariant series whose factors belong to $S_w (w \in \{0, 1, 2, 3, 4\})$.

We can observe that in near-ring-theory the classes S_i ($i \in \{0, 1, 2, 3, 4\}$) never coincide without further conditions while in ring-theory, for instance, S_1 and S_2 coincide.

In order to establish relationships between the classes S_w, let us state some results that concern the near-rings belonging to S_2. We recall that: A near-ring N is N_0-simple if it is without proper additive subgroups S such that $SN_0 \subseteq S$.

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(1) We observe that if N is zero-symmetric, N_0-semplicity and N-semplicity coincide.

Definition 2. A zero-symmetric near-ring N is *A-simple* if it is without non-zero N-subgroups H such that $HN = \{0\}$.

Theorem 1. *A near-ring N belongs to S_2 iff N is a zero-ring of prime order, a constant near-ring of prime order or an A-simple and strongly monogenic near-ring .*

Proof. Let N be an N_0-simple near-ring. The constant and the zero-symmetric parts are both N_0-subgroups of N, hence N is constant or zero-symmetric . By [2] and Ex.3.9 p.78 of [12] a constant near-ring is N_0-simple iff it is cyclic of prime order. If N is zero-symmetric either $nN = \{0\}$ for every $n \in \mathbb{N}$, and thus N is a zero-ring of prime order, or N is strongly monogenic and obviously A-simple. Conversely, if N is a zero-ring of prime order or a constant near-ring of prime order, then N is N_0-simple. Let N be an A-simple and strongly monogenic near-ring. Let us suppose that M is a proper N_0-subgroup of N. Since N is an A-simple near-ring, then $MN \neq \{0\}$ and since N is a strongly monogenic near-ring there is an element $h \in M$ such that $hN = N$. Since M is an N_0-subgroup, hN is contained in M, a contradiction. Thus N is N_0-simple. $\diamondsuit$

We observe that a zero-symmetric near-ring which is A-simple and strongly monogenic is Blackett simple ([4]).

Definition 3. A near-ring N is *strongly N_0-simple* if its subnear-rings belong to S_2.

We will call S_2^s the class of the strongly N_0-simple near-rings .

Theorem 2. *If N is an N_0-simple near-ring and every subnear-ring M of N satisfies the d.c.c. on the M-subgroups, then N is strongly N_0-simple.*

Proof. By Th.1, if N is a zero-ring of prime order or a constant near-ring of prime order, then N is strongly N_0-simple. Let N be an A-simple and strongly monogenic near-ring and let M be a subnear-ring of N with d.c.c. on the M-subgroups. Our aim is to show that M does not contain additive subgroups S so proving that $SM \subseteq S$. Let us suppose S to be a proper M-subgroup of M. Since N is A-simple then $SN \neq \{0\}$, thus there is an element $s \in S$ such that $sN = N$, given that N is strongly monogenic. Firstly we observe that $r(s) = \{0\}$ (where $r(s)$ is the right annihilator of the element s). In fact $r(s) \neq \{0\}$ implies $r(s)N \neq \{0\}$, because $r(s)$ is an N-subgroup of N and N is A-simple; thus $r(s)N = N$ and $N = sN = s[r(s)N] = \{0\}N = \{0\}$ and this is absurd. Moreover, since S is a proper M-subgroup of M, sM

is strictly contained in M. We set $M_1 = sM$ and consider sM_1. It is an M-subgroup of M strictly contained in M_1, in fact if $sM_1 = M_1$, it would be $ssM = sM$, that is $s(sM - M) = \{0\}$. Since $r(s) = \{0\}$, then $sM = M$ and this was previously excluded. In this way we obtain a chain $M_1 \supset sM_1 \supset s^2M_1 \supset \ldots$ which becomes stationary, due to d.c.c. on the M-subgroups. Since this is excluded, M is M-simple. ◇

Proposition 1. *If N is a wJ-near-ring, then N is a $(w-1)J$-near-ring.*

Proof. We can easily prove that $S_4 \subset S_3 \subset S_2 \subset S_1 \subset S_0$ and consequently that a wJ-near-ring is a $(w-1)J$-near-ring. ◇

Proposition 2. *The classes S_w $(w \in \{0,1,2,3,4\})$ are closed under homomorphisms and the classes S_w $(w \in \{3,4\})$ are closed under substructures.*

Proof. The near-rings belonging to S_3 and S_4 are without substructures and simple, so they do not have proper homomorphic images. Moreover, if $N' = \varphi(N)$ is a homomorphic image of N, each proper N'_0-subgroup (ideal) of N' derives from some proper N_0-subgroup (ideal) of N, thus $N \in S_2$ implies $N' \in S_2$ ($N \in S_0$ implies $N' \in S_0$). Moreover, if N is strongly monogenic and simple, then N' is strongly monogenic and simple, therefore $N \in S_1$ implies $N' \in S_1$. ◇

Hence, by Prop.6 of [1]:

Proposition 3. *The classes of the $3J$-near-rings and of the $4J$-near-rings are closed under substructures, homomorphic images and N_0-subgroups.*

We should observe that the classes $S_w(w \in \{0,1,2\})$ are not closed under substructures. In fact for example $\mathbb{Q} \in S_2$ but $\mathbb{Z} \notin S_0$. Therefore we cannot apply Prop.6 of [1] and, in fact, even if we can prove that S_2 is closed under N_0- subgroups, the class of the $2J$-near-rings is not closed under N_0-subgroups.

2-Jordan near-rings

The following Th.3, which provides a necessary and sufficient condition so that the class S_2 is closed w.r.t. substructures, uses the Th.1.33 of [11].

Let I be an ideal of a near-ring N and S a subnear-ring of N. Then $I \cap S$ is an ideal of S, I is an ideal of $I+S$ and $I+S/I$ is isomorphic to $S/I \cap S$.

Theorem 3. *A near-ring N has all its subnear-rings as $2J$-near-rings iff it contains an invariant series $N = N_1 \supset N_2 \supset \ldots \supset \supset N_n = \{0\}$ whose principal factors $N_i^!$ belong to S_2^s.*

Proof. Let N be a near-ring whose subnear-rings are $2J$-near-rings. So N is also a $2J$-near-ring. Hence let us consider an invariant series of N,

$$(\alpha) \qquad N = N_1 \supset N_2 \supset \ldots \supset N_n = \{0\}$$

whose principal factors belong to S_2. In order to show that the principal factors of (α) belong to S_2^s, we will show that every subnear-ring M of $N_i^!$ has the d.c.c. on the M-subgroups. Let M be a subnear-ring of $N_i^!$. Since M is a homomorphic image of a subnear-ring of N_i and consequently of N, by Proposition 2, it is a $2J$-near-ring. Therefore M has an invariant series $M = M_1 \supseteq M_2 \ldots \supseteq M_n = \{0\}$ whose factors belong to S_2. Hence these factors have the d.c.c. on the $(M_i^!)_0$-subgroups. By Th.1 and Ex a) of [1] we can deduce that M also has the d.c.c. on M-subgroups. Thus $N_i^!$ belong to S_2 and every subnear-ring M of $N_i^!$ has the d.c.c. M. We apply Th.2 and $N_i^! \in S_2^s$.

Conversely, let N be a near-ring with an invariant series $N = N_1 \supset \supset N_2 \supset \ldots \supset N_n = \{0\}$ whose principal factors $N_i^!$ belong to S_2^s. We can prove that the subnear-rings of N are $2J$-near-rings. Let M be a subnear-ring of N. We set $M_i = M \cap N_i$ and we obtain an invariant series of M : $M = M_1 \supseteq M_2 \supseteq \ldots \supseteq M_n = \{0\}$.

By the Theorem 1.33 of [11], $N_{i+1} + M_i/N_{i+1}$ is isomorphic to $M_i/N_{i+1} \cap M_i$ that coincides with M_i/M_{i+1}. Therefore $M_i^!$ is isomorphic to $N_{i+1} + M_i/N_{i+1}$ and the latter is a subnear-ring of $N_i^!$. Since $N_i^!$ belongs to $S_2^s, M_i^!$ belongs to S_2 and M is a $2J$-near-ring. ◇

Corollary 1. *The class of finite $2J$-near-rings is closed under substructures.*

Proof. It follows from Th.2 and 3, given that, in the finite case, the d.c.c. hold. ◇

In the following N will be a zero-symmetric near-ring.

Theorem 4. *If N is a near-ring with an A-simple and strongly monogenic ideal I such that N/I is a zero-ring of prime order, then $N = I \oplus J$ where $J = J_2(N)$.*[2]

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[2] $J_2(N)$ is the intersection of right annihilators of N_0-simple N-groups, see [12] p. 136.

Proof. Let I be a proper ideal of N, otherwise the thesis is trivial. Since N is zero-symmetric, I is an N-subgroup of N, therefore $IJ_2(N) = \{0\}$ and $J_2(N) \neq N$, $J_2(N) \neq I$ because I is A-simple. Moreover $J_2(N) \neq \{0\}$. In fact: if $J_2(N) = \{0\}$, then $J_2(I) = \{0\}$ and I is 2-semisimple with d.c.c. on the right annihilators. Hence I has a left identity e (see [2], [4], [12] p. 146) and by Pierce decomposition $N = r(e) + eN$. We observe that $r(e) \neq \{0\}$. In fact $r(e) = \{0\}$ implies $N = eN \subseteq I$ and this is excluded. Moreover N/I is a zero-ring, therefore $[r(e)]^2 \subseteq I$ and hence $[r(e)]^2 = \{0\}$. In this way $r(e)$ is a non trivial nilpotent N-subgroup of N and therefore $r(e) \subseteq J_2(N) = \{0\}$ (see [12] p. 153, [13]), a contradiction. Finally $I \cap J_2(N) = \{0\}$ because I is simple and $N = I + J_2(N)$ because N/I is of prime order. Hence $N = I \oplus J_2(N)$. ◊

The following theorem shows that, given a zero-symmetric near-ring with an invariant series whose factors are in S_2, it is possible to construct another invariant series whose factors are in S_2 such that the A-simple and strongly monogenic factors precede the zero-ring factors.

Theorem 5. *Let N be a $2J$-near-ring and $N = N_1 \supset N_2 \supset \ldots \supset N_n = \{0\}$ an invariant series whose principal factors are in S_2. If $N_i^!$ is a zero-ring and $N_{i+1}^!$ is an A-simple and strongly monogenic near-ring then there is an ideal M_{i+1} of N such that $N_i \supset M_{i+1} \supset N_{i+2}$, N_i/M_{i+1} is isomorphic to $N_{i+1}^!$ and M_{i+1}/N_{i+2} is isomorphic to $N_i^!$.*

Proof. Considering the near-ring N_i'', we set $I = f_i''(N_{i+1})$. Given that $N_i''/N_{i+1}^!$ is isomorphic to $N_i^!$ we have N_i''/I isomorphic to $N_i^!$. Therefore N_i''/I is a zero-ring of prime order and I is A-simple and strongly monogenic because it is isomorphic to $N_{i+1}^!$. Hence, by Th.4, $N_i'' = I \oplus J$ where $J \simeq N_i^!$ and therefore $N_{i+1}^! \simeq N_i''/J$. We set $M_{i+1} = (f_i'')^\circ(J)$, that is M_{i+1}/N_{i+2} is isomorphic to $N_i^!$. Obviously M_{i+1} is an ideal of N_i and $N_i/M_{i+1} \simeq (N_i/N_{i+2})/(M_{i+1}/N_{i+2}) \simeq N_i''/J \simeq I \simeq N_{i+1}/N_{i+2} = N_{i+1}^!$. Hence M_{i+1} is a maximal ideal of N_i.

Now we can show that M_{i+1} is an ideal of N: the near-ring N_{i+1} is an ideal of N, M_{i+1} is an ideal of N_i, hence $N_{i+1}M_{i+1} \subseteq N_{i+1} \cap M_{i+1}$. Moreover $N_{i+1} \cap M_{i+1} = N_{i+2}$. In fact if $x \in N_{i+1} \cap M_{i+1}$, then $x + N_{i+2} \in N_{i+1}^! \cap J = \{0\}$ and this implies that $x \in N_{i+2}$. Thus $N_{i+1} \cap M_{i+1} \subseteq N_{i+2}$. Obviously $N_{i+2} \subseteq N_{i+1} \cap M_{i+1}$, therefore $N_{i+1} \cap M_{i+1} = N_{i+2}$. We now set $(N_{i+2} : N_{i+1})_N = \{m \in N / N_{i+1}m \subseteq N_{i+2}\} = H$ which is an ideal of N (see [11]). We obtain $M_{i+1} \subseteq$

$\subseteq H \cap N_i$ and $H \cap N_i$ is strictly enclosed in N_i, otherwise it would be $N_{i+1}\, N_i \subseteq N_{i+2}$ and hence $N_{i+1}\, N_{i+1} \subseteq N_{i+2}$, but N'_{i+1} is A-simple and this is excluded. Hence $M_{i+1} = H \cap N_i$. Thus M_{i+1}, as intersection of two ideals of N, is an ideal of N. $\diamondsuit$

Theorem 6. *A non nilpotent $2J$-near-ring N, has the radical $J_2(N)$ nilpotent and the factor $N/J_2(N)$ is a direct sum of A-simple and strongly monogenic near-rings.*

Proof. By Th.5, if N is a zero-symmetric $2J$-near-ring, we can construct a new invariant series $N = N_1 \supset N_2 \supset \ldots \supset N_n = \{0\}$ whose factors are in S_2, such that, if N'_i is A-simple and strongly monogenic and N'_j is a zero-ring, then $i < j$. We set $h \in I_n$, the smallest index such that N'_h is a zero-ring. Obviously N_h is nilpotent. Therefore $N_h \subseteq J_2(N)$. Moreover, if $N_h \neq N$, the near-ring N/N_h contains an invariant series whose factors are N-simple and hence 2-semisimple. By Ex. f) of [1], N/N_h is 2-semisimple and therefore $J_2(N) \subseteq N_h$. Hence $J_2(N) = N_h$ and the radical $J_2(N)$ is nilpotent. In this way $N/J_2(N)$ has an invariant series satisfying the hypotheses of Th.4 of [1], thus $N/J_2(N)$ is the direct sum of A-simple and strongly monogenic near-rings. $\diamondsuit$

The analogous, in ring-theory, brings us to the famous theorem of Artin-Noether. In fact, rings with an invariant series whose factors are in S_2, are rings with an invariant series whose factors are without right ideals(3) and hence are either fields or zero-rings. Thus in a ring A satisfying the hypotheses of Th.6 the Jacobson radical $J(A)$ is nilpotent and the factor $A/J(A)$ is a direct sum of fields.

Corollary 2. *Let N be a $2J$-near-ring. Then $\mathcal{P}(N) = \eta(N) = J_0(N) = J_1(N) = J_2(N)$.*(4)

Proof. It can be easily demonstrated, since N has the d.c.c. on the N-subgroups and $J_2(N)$ is nilpotent (see 5.61 p. 162 of [12]). $\diamondsuit$

If N is a finite near-ring, we obtain:

Corollary 3. *Let N be a finite near-ring such that $N \neq J_2(N)$. Then:*

1. *If N is a $2J$-near-ring and the A-simple factors present in a principal series are planar, then the additive group $(N/J_2(N))^+$ is nilpotent;*
2. *If N is a $3J$-near-ring, the additive group $(N/J_2(N))^+$ is abelian.*

Proof. The group $(N/J_2(N))^+$ is a direct sum of finite groups sup-

(3) A ring having an invariant series whose factors are in S_2, is right artinian.

(4) For the definitions of $\mathcal{P}(N)$, $\eta(N)$ and $J_v(N)$ ($v \in \{0,1,2,\}$) see [9], [11], [12].

porting planar near-rings. Therefore, as shown in [3], $(N/J_2(N))^+$ is nilpotent.

If N is a $3J$-near-ring, the factors of the invariant series are without proper subnear-rings. Therefore, as proved in [6], (see also [7]) they are p-singular[(5)] and therefore their additive group is elementary abelian, because they are simple. Thus $(N/J_2(N))^+$, being a direct sum of elementary abelian groups, is abelian. ◇

4-Jordan near-rings

In this section we will study the $4J$-near-rings with particular reference to the nilpotent case. We recall that a near-ring N is *nilpotent* if there is an index $n \in \mathbb{N}$ such that $N^n = \{0\}$. We will call $g(N)$ the least $n \in \mathbb{N}$ such that $N^n = \{0\}$ and $\dim(N)$ the length of an invariant series whose factors are in S_4.

Theorem 7. *A near-ring N with an invariant series $N = N_1 \supset N_2 \supset \supset \ldots \supset N_n = \{0\}$ and whose factors are in S_4 is nilpotent iff $N^s \subseteq N_s$, for every $s \in I_n$.*

Proof. Let N be a nilpotent $4J$-near-ring. *We will show that, for every $i \in I_n$, $NN_i \subseteq N_{i+1}$.* If $NN_i \not\subseteq N_{i+1}$, there is an element $a \in N$ such that $aN_i \not\subseteq N_{i+1}$. Since aN_i is a subnear-ring of N_i and N_i/N_{i+1} is of prime order, $(aN_i + N_{i+1})/N_{i+1}$ is not a proper subnear-ring of N_i/N_{i+1}. Therefore, either $aN_i + N_{i+1} = N_{i+1}$ or $aN_i + N_{i+1} = N_i$. Given that $aN_i \not\subseteq N_{i+1}$, we have:

$$(\alpha) \qquad aN_i + N_{i+1} = N_i$$

and $a^hN_i = a^{h+1}N_i + a^hN_{i+1}$. Let h' be the smallest integer such that $a^{h'}N_i \subseteq N_{i+1}$. This h' exists and it is $h' > 1$ because otherwise, for every $t \in \mathbb{N}$, it would be $a^tN_i + N_{i+1} = N_i$ and since N is nilpotent, it would be $N_{i+1} = N_i$ and this is excluded. Therefore, by (α), we obtain $a^{h'}N_i + a^{h'-1}N_{i+1} = a^{h'-1}N_i$, hence $a^{h'-1}N_i \subseteq N_{i+1}$ in contrast to the hypothesis stating that h' is the smallest integer so that $a^{h'}N_i \subseteq N_{i+1}$. Thus $NN_i \subseteq N_{i+1}$ and consequently $N^s \subseteq N_s$ for every $s \in I_n$. The converse is trivial. ◇

(5) For the definition of p-singular near-ring see [6].

Corollary 4. *If N is a nilpotent 4J-near-ring,* $g(N) \leq \dim(N)$.
Proof. It is a consequence of Th.7. ◇

We can characterize the case in which $g(N) \doteq \dim(N)$.

Theorem 8. *Let N be a nilpotent 4J-near-ring and let $N = N_1 \supset \supset N_2 \supset \ldots \supset N_n = \{0\}$ a series whose factors are in S_4. The length of the chain and the nilpotence index of N coincide iff $N_i = (N_{i+1} : N)_N$ for every $i \in I_{n-1}$.*

Proof. We set $M_i = (N_{i+1} : N)_N = \{n \in N/Nn \subseteq N_{i+1}\}$. Let $g(N) = dim(N) = n$. By Th.7, we have $NN_i \subseteq N_{i+1}$ and hence $N_i \subseteq \subseteq M_i$. If N_i is strictly contained in M_i, the series $N \supset M_i \supset N_i \supset \{0\}$ will be refinable (by Jordan-Hölder theorem) in a principal series where $M_i = \overline{N}_j$ with $j \leq i$. By Th.7, $N^j \subseteq \overline{N}_j$ and hence $N^j \subseteq M_i$. Therefore $N^{j+1} \subseteq NM_i \subseteq N_{i+1}$. Hence $N^{j+1+(n-i-1)} = N^{n-(i-j)} = \{0\}$. Given that $g(N) = n$, we obtain $i = j$, that is $M_i = \overline{N}_j = N_i$.

Conversely, let us suppose $N_i = M_i$ for every $i \in I_{n-1}$ and $g(N) = h$. Then $N^h = \{0\}$, therefore $N^{h-1} \subseteq (0 : N)_N = N_{n-1}$, in fact N_{n-1} is the right annihilator of N because $N_{n-1} = M_{n-1} = (N_n : N)_N$. Analogously $N^{h-2} \subseteq (N_{n-1} : N)_N = N_{n-2}$ and so on. After a finite number of steps we get $N \subseteq N_{n-h+1}$, thus $N = N_{n-h+1}$ and $n = h$. ◇

Finally:

Theorem 9. *If N is a nilpotent 4J-near-ring such that $g(N) = \dim(N)$, then $|N| = p^\alpha$, (p prime).*

Proof. We can prove this theorem by induction on $g(N)$. If $g(N) = 2$, $N = N_1 \supset N_2 = \{0\}$ is the principal series required and hence $|N| = p$. Let us suppose the theorem proved for dim $(N) = n - 1$ and let $N = N_1 \supset N_2 \supset \ldots \supset N_n = \{0\}$ be a series of N whose factors are in S_4. Then $|N/N_{n-1}| = p$ and we can suppose $|N_{n-1}| = q$ (q prime). By Th.7, $N^{n-2} N = N^{n-1} \subseteq N_{n-1}$, therefore, for every $m \in \in N^{n-2}$, $mN \subseteq N_{n-1}$ and given that N_{n-1} is of prime order, either $mN = \{0\}$ or $mN = N_{n-1}$. If $mN = \{0\}$, for every $m \in N^{n-2}$, then $N^{n-1} = \{0\}$ and this is excluded, thus $mN = N_{n-1}$ for some $m \in N$. Considering now the left translation $\gamma_m : N \to mN$, we obtain an endomorphism of N^+ whose kernel is $r(m)$, the right annihilator of m and whose image is N_{n-1}. Therefore $|\text{im}\, \gamma_m| = |N/\ker \gamma_m|$ that is $q = |N/\ker \gamma_m|$. Given that $\ker \gamma_m = r(m) \supseteq r(N) = N_{n-1}$, either $|\ker \gamma_m| = q$ or $|\ker \gamma_m| = q^\beta$. Thus: $q = qp^\alpha/q^\beta$ and this implies $q^\beta = p^\alpha$, hence $p = q$. ◇

References

[1] BENINI, A. and PELLEGRINI, S.: Invariant series in universal algebras, Ω-groups and near-rings , *Contribution to General Algebra* **7**, Wien (1991), 29 – 36.

[2] BETSCH, G.: Struktursätze für Fastringe, Diss. Univ. Tübingen, 1963.

[3] BETSCH, G. and CLAY, J.R.: Bloch designs from Frobenius groups and planar near-rings, Proc. Conf. finite groups (Park City Utah), Acad. Press. (1976), 473 – 502.

[4] BLACKETT, D.W.: Simple and semisimple near-rings, *Proc. Amer. Math. Soc.* **4** (1953), 772 – 785.

[5] FERRERO COTTI, C.: Sugli stems in cui il prodotto è distributivo rispetto a se stesso, *Riv. Mat. Univ. Parma* (3) **1** (1972), 203 – 220.

[6] FERRERO, G.: Struttura degli stems p-singolari, *Riv. Mat. Univ. Parma* (2) **7** (1966), 243 – 254.

[7] FERRERO, G.: Classificazione e construzione degli stems p-singolari, *Ist. Lomb. Accad. Sci. Rend. A.* **102** (1968), 597 – 613.

[8] KAARLI, K.: A new characterisation of semi-primary near-rings, S.Benedetto del Tronto, 1981, 83 – 94.

[9] KAARLI, K.: Radicals of near-rings, *Tartu Riikl. Ül Toimetised* **390** (1976), 134 – 171.

[10] MASON, G.: Solvable and nilpotent near-rings modules, *Proc. Amer. Math. Soc.* **40** (1973), 351 – 357.

[11] MELDRUM, J.D.P.: Near-rings and their links with groups, Research Notes in Mathematics **134**, Pit. Adv. Pub. Prog.

[12] PILZ, G.: Near-rings, 2nd. ed., North Holland Math. Studies **23**, Amsterdam, 1983.

[13] RAMAKOTAIAH, D.: Radicals for near-rings, *Math. Z.* **97** (1967), 45 – 56.